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PROVISION
ON APPLICATION OF MATHEMATIC STATISTICS METHODS FOR RECORD AND CONTROL OF NUCLEAR MATERIALS

(RB-066-11)

Abbreviations

This document involves following abbreviations:

|  |  |  |
| --- | --- | --- |
| IBP | - | Inter-balance period  |
| MM | - | Method of Measurements |
| MBA | - | Material balance area |
| ID | - | Inventory difference |
| KMP | - | Key measurement point |
| ACM | - | Access-control means |
| MSD | - | mean square deviation |
| AASL | - | Actually available Stock List |
| FA | - | Fuel assembly |
| FE | - | fuel element |
| II | - | Inventory item |
| PhI | - | Physical inventory |
| NM | - | Nuclear material |

I. General

1. The Provision on use of mathematical statistics methods for accounting and control of nuclear materials (hereinafter – the Provision) is included in the safety guides and intended as guidelines and represents no normative legal act.

2. This Provision includes the guidelines of the Federal Environmental, Industrial and Nuclear Supervision Service on use of mathematical statistics methods for record and control of nuclear materials.

3. This Provision is considered to be used for document preparation of the institution involved in statistical processing of measurement results of quantity and composition of nuclear material in each material balance area and quantity and composition of nuclear material, when it is transferred between material balance areas and when drawing up balance sheet of nuclear material in material balance areas.

4. The Provision does not cover all existing methods of mathematical statistics. It presents most applicable mathematical statistics methods used in practice of NM record and control, and guidelines for their application to solve tasks for NM record and control, such as:

establish compliance of the actual II parameters with the existing inventory data;

determine amount of confirmatory measurements and analysis of the difference between inventory and confirmatory measurements results of the quantitative NM, II and products parameters;

determine amount of random sampling, when checking seals;

analysis of NM data discrepancies between the sender and recipient organizations;

 taking inventory and confirmation measurements of actually available nuclear material considering measurement error;

assessment of unmeasured losses of nuclear material and its error;

determine inventory difference error, statistical analysis of ID significance.

5. The guidelines of the Provision apply only to nuclear material that is registered and controlled as an Inventory Item.

6. This Provision includes practical examples of possible use of mathematical statistics methods.

II. Guidelines for establishing compliance of the actual inventory item parameters with existing inventory data

7. It is advised to determine whether the actual II parameters correspond to existing inventory data in organizations by comparing the measurement results of II parameters with the inventory data For example, when determining whether the measurement results of II parameters do not correspond to the inventory data, and also in organizations, where new inventory items are formed. It is advised to perform the inventory measurements in accordance with specially developed method of measurement MM, which provides either single measurement of the checked II parameter, or perform several simultaneous measurements of this parameter. When the MM specifies single measurements of the checked II parameter, it also defines the procedure to obtain guaranteed limits of the measurement error. It is advised to consider coincidence of the measurement result with the inventory data within these limits as correspondence of the actual value of the II characteristic to the existing inventory data.

8. It is advised to perform statistical processing of single measurement results to confirm quantity of nuclear material in inventory item sampling on the basis of certain characteristics.

Thus, for example, when determining the weight of nuclear material, the average weight of nuclear material in sampling of homogeneous inventory item (produced according to the same manufacturing procedure specification) can be calculated using following formula:

$\overline{m}= \frac{\sum\_{i=1}^{n}m\_{i}}{n}$ (1)

where:

mi - value of NM weight in individual inventory item sample;

n - number of inventory items in the sample.

To determine the mi values using direct or indirect methods, it is advised to use certified method of measurements, which define order of the measurement and guaranteed error of the result.

It is advised to determine the sample variance of NM weight in inventory item according to following expressions:

$S\genfrac{}{}{0pt}{}{2}{m}= \frac{1}{n-1} \sum\_{i=1}^{n}(m\_{i}- \overline{m})^{2}$ (2)

or

$S\genfrac{}{}{0pt}{}{2}{m}= \frac{1}{n-1} \left[\sum\_{i=1}^{n}m\genfrac{}{}{0pt}{}{2}{i}- \frac{1}{n} (\sum\_{i=1}^{n}m\_{i})^{2}\right]$, (2.1)

It is advised to calculate the mean square deviation and coefficient of variation for the sampling by formulas:

$S\_{m}= \sqrt{S\genfrac{}{}{0pt}{}{2}{m}}$ (3)

and

$nu= \frac{S\_{m}}{\overline{m}} x 100\%$. (4)

Calculation of these characteristics is advised to perform using integrated statistical packages. In case of significant discrepancy with inventory data, when measuring quantity of nuclear material in products using available measurement method, it is advised to make sure that it is appropriate to continue using the mentioned characteristics.

For this purpose it is advised to check proper statistical hypotheses.

9. To check the hypotheses, we advise to use approach based on deliverance of statistical conclusion for the nuclear material quantity the inventory item, using two types of hypotheses: the null hypothesis, which is that the actual quantity of nuclear material equals to the declared number, and the alternative hypothesis, which is that the quantity of NM in the inventory item differs from the declared number.

When testing the hypotheses, there may be an error involving rejecting the null hypothesis, when it is true (error of the 1st kind), and error consisting in accepting the null hypothesis, when alternative hypothesis is true (error of the 2nd kind).

The probabilities of errors of the 1st and 2nd kind are denoted as alpha and beta, respectively.

10. Checking consent of the experimental distribution with the normal law is advised to perform in accordance with the Rules of check of experimental and theoretical distribution of the consent (guidelines for standardization R 50.1.037-2002). However, depending on the number of measurements, it is advised to use different criteria.

For example, when the number of measurements exceeds fifty, we advise using the omega criterion $\frac{2}{2}$.

When calculating omega, a tabular display method is advised, where the values, for example, of content of each isotope xi (i = 1,...,n) are arranged in ascending order and ordering results (xj, j = 1,..., n) are entered in the first column of the tables. The second column of the tables contains values of distribution function of the predicted distribution being tested, F(xj ), j = 1,...,n. The third column of the tables contains values ln F(xj ), j = 1,...,n. The forth one contains values (2j - 1) / (2n), j = 1,...,n. The fifth one contains products of values in columns (3) and (4) for j = 1,...,n. The sixth column contains difference between one and values in column (4) for j = 1,...,n. The seventh column contains difference between one and values in column (2) for j = 1,...,n. The eighth column contains values ln [1 - F(xj )], j = 1,...,n. The ninth column contains products of values in columns (6) and (8) for j = 1,...,n. The tenth column contains the sum of values in columns (5) and (9) for j = 1,...,n.

The tables are used to calculate values of omega $\frac{2}{n}$ = -n - 2 SUM {[(2j - 1) / (2n)] ln F(xj ) + [1 - (2j - 1) / (2n)] ln [1 - F(xj )]}. Here summation over j = 1,...,n. The calculated omega values$\frac{2}{n}$ are compared with critical value for significance level of 0.05, given in the Rules of check of experimental and theoretical distribution of the consent (guidelines for standardization R 50.1.037-2002).

11. Checking stochastic independence of the results of parallel definitions is advised to perform on the basis of criterion of consecutive difference squares ratio. For this purpose, the value for each n simultaneous measurements is calculated:

gamma(n) = g2 (n) / s2 (n), (5)

where:

$g^{2}\left(n\right)= \left[\sum\_{i=1}^{n-1}(x\_{i+1}- x\_{i})^{2}\right] / \left[2(n-1)\right]$,

$s^{2}n= \left[\sum\_{i=1}^{n}(x\_{i}- \overline{x})^{2}\right] / (n-1)$, $\overline{x}= \left(\sum\_{i=1}^{n}x\_{i}\right) / n$.

In case of gamma(n) > gamma'0,05 (n), where gamma'0,05 (n) = 1 + and 0,05 / √n - 1, and 0,05 - is a tabulated value, the hypothesis on stochastic independence of results of parallel definitions is not rejected.

12. It is advised to evaluate insignificant discrepancy between results of parallel definitions (xi, i = 1,...,n) for each series of n measurements with confidence coefficient of 0.95 using the inequation:

$∣x\_{i}- \overline{x}∣ < beta^{В}S\_{r}\overline{x}$*,* (6)

where upper bound of relative MSD of measurement results is beta BSr = epsilon √n / (2x), confidence bounds (interval) of the random error of measurement result is epsilon = t(n-1); 0.95 Sx / √n-1, the arithmetic mean

$\overline{x}= \left(\sum\_{i=1}^{n}x\_{i}\right) / n$, $CKO S\_{x}= \sqrt{(\sum\_{i=1}^{n}(x\_{i}- \overline{x})^{2}) / (n-1)}$.

t(n-1);0,95 is the value of Student's coefficient with (n - 1) degrees of freedom with confidence coefficient of 0.95. When calculating the beta coefficient value, existing standards and guidelines in this field are used. If the inequation is met, there is no reason to exclude the i-th value from the analysis as irregular one.

13. To check whether the difference in variance is insignificant according to the Cochran test, it is advised to calculate the values for each isotope (element) of samples of each content:

$G\_{max}=S\genfrac{}{}{0pt}{}{2}{max} / \sum\_{i=1}^{m}S\genfrac{}{}{0pt}{}{2}{i}$. (7)

Here $S\genfrac{}{}{0pt}{}{2}{i}= \sum\_{j=1}^{N}(x\_{j}- \overline{x})^{2} / (N-1)$, and $S\genfrac{}{}{0pt}{}{2}{max}$ - greatest out of $S\genfrac{}{}{0pt}{}{2}{i}$. Calculation of $\overline{x }$ shall be made by following formula: $\overline{x} = \sum\_{j=1}^{N}x\_{j} / N$, N is the number of parallel determinations of the content of each isotope (element) in each sample, and m is the number of samples of each content.

To check whether the variance difference is insignificant, the calculated Gmax values are compared with the critical value of Gmu (N - 1, m) for the significance level of 0.05, which is determined in accordance with existing standards and guidelines in this field.

14. When investigating the homogeneity of samples of the element (isotope) content for each content for each element (isotope), the sum of squares of deviations for determination results within the samples (samples here are the samples with the same content) $SSе = \sum\_{n=1}^{N}\sum\_{j=1}^{J} (x\_{nj}-\overline{x\_{n}})^{2}$and between arithmetic average values for samples$SSn = J \sum\_{n=1}^{N}(\overline{x\_{n}}- \overline{x})^{2}$ is calculated. Here $\overline{x\_{n}}$ is arithmetic mean of J parallel definitions (xnj, j = 1,..., J) of each element (isotope) in the nth sample of investigated content, and $\overline{x}$ is the arithmetic mean of all xnj NJ definitions. Then the sampling mean squares of deviations of results within the samples $\overline{SS\_{e}}$ = SS / (N (J - 1)) and between the samples $\overline{SS}$n = SSn / (N - 1) are calculated. For indivisible MSD samples that characterize nonhomogeneity of the material by content of each element (isotope) in the samples of each element (isotope) content calculation is performed by formula:

$sigma\_{n}= \sqrt{\overline{SS\_{n}} / } J$*.*

Example of comparing of two algorithms for spectra processing is given in Appendix 1 to this Provision.

III. Determining amount of confirmatory measurements and the inventory item for measurements. Evaluation of difference between the results of inventory and confirming measurements

15. Where possible, it is advised to conduct confirmatory measurements of all nuclear material that is contained in material balance area. Otherwise, it is advised to measure inventory item randomly selected from its total amount in material balance area. It is advised to use differentiated approach that takes into account influence of three factors: NM category, level of application of ACM for it, and weight of nuclear material in inventory item.

16. When implementing the differentiated approach for selection of inventory item for the confirmatory measurements, it is advised to distribute them by strata (aggregates of individual inventory items with identical or similar physical characteristics and chemical composition of nuclear material) and make a list of inventory items included in each stratum. The sample volume is set by two values: G - by threshold quantity of nuclear material in weight units and P - by probability of detecting a shortage/excess of the threshold NM quantity.

17. The threshold amount of G is advised to be determined depending on nuclear material category.

18. The probability of detecting shortage / excess of P is advised to be determined according to Table 1.

Table 1

PROBABILITY OF DETECTING SHORTAGE/EXCESS OF THE THRESHOLD NM QUANTITY FOR CALCULATING THE SAMPLE VOLUME OF CONFIRMING MEASUREMENTS

|  |  |
| --- | --- |
| Scope of ACM application to the nuclear material | Detection probability, Р %, minimum |
| For categories 1, 2 and 3 | For category 4 |
| Only seals | 50 | 30 |
| Only surveillance system | 50 | 30 |
| At the same time, two types of various ACM | 25 | 9 |
| At the same time, m types of various ACM | 100 х (0,5)m | 100 х (0,3)m |

19. If nuclear materials are presented in the form of inventory items with their integrity in the material balance area within the IBP assured by their design (for example unibody fuel assemblies or disks, blocks, items with nuclear materials covered with cladding so that nuclear materials could not be withdrawn without loss of the cladding integrity) and enabling confirmation (by visual examination, comparison of gamma- or ultrasonic scanning images for weld joints) the detection parameters similar to the ones applied to inventory items with seals shall be used to calculate the confirmatory measurement sample volume for these nuclear materials. Here it is advised to take into account availability of documentary evidence of integrity of unibody inventory items and absence of unauthorized access, as well as the actual testing of ACM during physical inventory.

20. The sample volume (n) can be calculated in accordance with Appendix 2 to this Provision by formula:

n = [N (1 - (1 - P)1/[G/x])]+, (8)

where:

N - number of inventory items in stratum;

x - average weight of nuclear material in one inventory item;

[ ]+ - in the formula means rounding to the nearest larger integer.

21. Numbers of inventory items to be measured can be randomly selected from the list of inventory items to be checked for each stratum.

22. The random selection is advised to be performed using a random number generator with preliminary assignment of a sequence number from 1 to n to each inventory item of the tested stratum.

23. After measuring the quantitative parameters of nuclear materials, inventory items and products, it is advised to analyze the difference between results of inventory and confirming measurements.

For this purpose we advise to calculate the check tolerance value using following formula:

$K=2,58 \sqrt{sigma\genfrac{}{}{0pt}{}{2}{conf.}+sigma\genfrac{}{}{0pt}{}{2}{inv.}}$. (9)

Here sigmaconf. - mean square deviation of confirming measurements, and sigmainvent. - mean square deviation of inventory measurements.

24. When performing measurements of inventory item in several strata, when it is necessary to combine the measurement results, it is advised to test hypotheses for equality of average values and variances for normally distributed random variables. This may be necessary for combining several strata, or for splitting one stratum into several strata to get representative results. Example of practical solution to this problem is given in Appendix 3 to this Provision.

Two sample variances can be compared using Fischer F. test. For this purpose, find ratio of the larger sample variance to the smaller one.

If the ratio of sample variances $F= \frac{S\genfrac{}{}{0pt}{}{2}{1}}{S^{\frac{2}{2}}} at S\genfrac{}{}{0pt}{}{2}{1} >S\genfrac{}{}{0pt}{}{2}{2}$ is greater than or equal $F\_{1}- \frac{alpha}{2}$ with the number of degrees of freedom n1–1 and n2-1 where n1, n2 are sample volumes and alpha is the level of significance, it is advised to accept the hypothesis of inequality of two general variances, and therefore it is impossible to combine samples. If the condition is met, it is advised $F < F\_{1}- \frac{alpfa}{2}$ to accept the hypothesis of equality of general variances, and proceed to checking the equality of sample averages.

It is advised to compare the average values of normally distributed random values using the Student's test, which values are given in analysis of characteristics of the sample distribution (Appendix 3 to this Provision). For this purpose, in case of proven equality of sample variances, the summary variance is calculated according to the expression:

$S^{2}= \frac{\left(n\_{1}-1\right) S\genfrac{}{}{0pt}{}{2}{1}+\left(n\_{2}-1\right) S\genfrac{}{}{0pt}{}{2}{2}}{n\_{1}+ n\_{2}-2}$ (10)

and value t according to the expression:

$t= \frac{\overline{m\_{1}} - \overline{m\_{2}}}{S \sqrt{\frac{1}{n\_{1}}+ \frac{1}{n\_{2}}}}$. (11)

If |t| < tk, 1-alpha/2, then it can be accepted hypothesis of equality of mean values. Here alpha represents significance level, and k represents the number of degrees of freedom defined by expression k = n1 + n2-2. In case of |t| >= t k,1-alpha/2

$\overline{m\_{1}}/ = \overline{m\_{2}}$.

The recommended procedure for applying the acceptance tolerance and examples of comparing of two sample variances and averages, are given in Appendix 4 to this Provision.

IV. Determination of random sample volume, when checking seals;

25. To determine the sample volume of seals to be checked, it is advised to calculate the value n = [N (1 - (1 - 0,95) 1/(0,05 N))]+; where N is the number of installed seals in material balance area. Here, [ ]+ also means rounding to the nearest larger integer. The values of the sample volume for different values of N are given in Appendix 2 to this Provision.

26. The numbers of seals subject to verification are advised to choose at random.

V. Analysis of data discrepancies between the sender and receiver organizations

27. To analyze discrepancy between NM data for sender and receiver organizations, it is advised to determine acceptable limits of this discrepancy:

DELTAo-p = 2,58 х sigmao-p, (12)

where sigmao-p is mean square deviation of the weight measurement results using the sender's and receiver's scales. In general case:

$sigma\_{o-p}= \sqrt{R\genfrac{}{}{0pt}{}{2}{send}+S\genfrac{}{}{0pt}{}{2}{send}+R\genfrac{}{}{0pt}{}{2}{rec}+S\genfrac{}{}{0pt}{}{2}{rec}}$, (13)

where:

$R\genfrac{}{}{0pt}{}{2}{send}$ and $R\genfrac{}{}{0pt}{}{2}{rec}$ are values of the random error components of sender's and receiver's scales, respectively, and

$S\genfrac{}{}{0pt}{}{2}{send}$ and $S\genfrac{}{}{0pt}{}{2}{rec}$ are values of the systematic error components of sender's and receiver's scales, respectively.

In practice, the technical documentation for scales, as a rule, shows the full error, reduced to confidence interval of 0.99, which already includes both systematic and random error components (for example: in absolute units, DELTA = +/- 0.1 g or in relative delta = +/- 0.1%). In this case, the calculation is made by formula:

$sigma\_{o-p}= \sqrt{DELTA\genfrac{}{}{0pt}{}{2}{send}+DELTA\genfrac{}{}{0pt}{}{2}{rec}}$ (14)

or

$sigma\_{o-p}= \sqrt{(m delta\_{send})^{2}+ (m delta\_{rec})^{2}}$ (15)

If data for sender's scales is not available, the corresponding errors are assumed to be zero and permissible limits of discrepancy between sender's and receiver's NM data will be more "strict", accordingly. Examples of calculating of sigmao-p and an algorithm for evaluating significance of systematic discrepancies, are given in Appendix 5 to this Provision.

28. If there is a discrepancy between NM data for the sender and receiver organizations |msend-mrec| <= DELTAo-p (here msend is weight of NM according to the sender, and mrec is weight of NM according to the receiver), then it is advised to register the nuclear material in the receiver organization according to the sender's organization.

VI. Carrying out inventory and confirming measurements of the actual available quantity of nuclear material with consideration of measurement errors

29. In recording and control of nuclear materials, the most common task is comparison results of NM confirming measurements with inventory data. This task is described in section III of this Provision. However, sometimes there is a need to analyze data for nuclear material batch (section V of this Provision) and NM strata. In fact, the actual available quantity of nuclear material in stratum of N inventory items with weight Mk, k = 1,..., N can be determined as follows:

$I= \sum\_{k=1}^{N}M\_{k}=N \overline{M}$, (16)

where $\overline{M}$ is the average inventory value of inventory item weight.

30. When estimating the actual available quantity of nuclear material in the sample {$\hat{m\_{i}}$, i = 1,..., n} from bounded collection, where $\hat{m\_{i}}$ = mi + DELTA mi, mi is the true value of NM weight for the i-th inventory item, and DELTA mi is the uncertainty of this value, it is advised to consider at least three DELTA m components:

- the first (Delta) is a function of the measured weight, the sample characteristics that affect calibration and measurement method (for repeated measurements, the Delta behaves as a systematic error). The effect of delta on measured values is minimized by building calibration curves. In fact, it is not possible to completely exclude this component and each measurement has systematic deviation that depends on weight and other factors that affect the measurement result. Ideally, this component is already included in variance;

- second; this is a function of assessment of calibration curve, and coefficients of the latter usually depend on the measured weight. When two different inventory items are measured, this difference causes the measurement covariance, and the covariance in turn, depends on weights of these inventory items;

- third; epsilon is a random error associated with counting statistics. It takes independent values for repeated measurements of the same inventory item and for different inventory items as wel. In general, epsilon is a function of the measured weight.

Samples from the stratum are usually non-repetitive. Random values {mi, i = 1,..., n} are distributed equally, but unlike the case of sample from infinite entire assembly, they are not independent. This dependence is determined by covariance, which in this case equals to:

$cov\left(m\_{i}, m\_{j}\right)= sigma^{2} / (N-1)$. Here $sigma^{2}=(\sum\_{k=1}^{N}(M\_{k}- \overline{M})^{2}) / N$. (17)

The relationship between sample and stratum characteristics without consideration of the measurement error can be described as follows:

$E\left(\overline{m}\right)=E \left(m\_{i}, i=1, …, n\right), Var\left(\overline{m}\right)=\left(N-n\right)sigma^{2} / (\left(N-1\right)n)$*,*

$E((\sum\_{i=1}^{N}(m\_{i} - \overline{m})^{2}) / (n-1) )=N sigma^{2}/ (N-1)$.

Here E(x) and Var(x) are the mathematical expectation and variance of x, respectively.

31. The estimate of actual available quantity of nuclear material tends to the true value, when reducing the amount of systematic deviations that would make the measurement of each inventory item stratum:

$Var\left(\hat{I}\right)= N^{2} \left(N-n\right) \hat{\hat{sigma}\frac{2}{а} / ((N-1)} n)+ N^{2} \left(\sum\_{i}^{n}\sum\_{j}^{n}\hat{V\_{ij}}\right)/ n^{2}$ - variance of assessment.

Here $\hat{sigma}\genfrac{}{}{0pt}{}{2}{а}=\left(N-1\right)/ (N\left(n-1\right)) \left\{\sum\_{i=1}^{n}(\hat{m\_{i}} - \hat{<m>})^{2}- \sum\_{i}^{n}\hat{V\_{ii}}+(\sum\_{i}^{n}\sum\_{j}^{n}\hat{v\_{ij}}) / n \right\}$, where <m> is the average value $\hat{m\_{i}}$, i = 1,..., n, and $\hat{V\_{ii}}$, $\hat{V\_{ij}}$ are estimates of elements of the measurement error variance matrix. Such estimates of conventional values of variance and covariance depend on weight and serve as estimates of mathematical expectations for the sample of these variance and covariance values. The latter allows to work with samples from a finite collection with consideration of measurement errors, without limiting yourself to specific error model. The conventional values of variance and covariance are usually found in description and software of most up-to-date measurement systems.

The assessment of actual available quantity of nuclear material in stratum's inventory item tends to the true value, when the amount of systematic deviations that would be made by measuring each stratum's inventory item is reduced. To minimize amount of systematic deviations, the more thorough calibration is advised, considering dependence of systematic deviations on the measured weight and other factors affecting the measurement as well.

The above is true, when sufficiently close inventory item parameters are available. In case of significant differences in inventory item parameters, absolute weight values shall not be used, but discrepancies between these values obtained as a result of inventory and confirming measurements. In this case, the total amount of data discrepancy shall be assessed.

VII. Calculation of inventory difference error.

Statistical analysis of inventory difference significance

32. The inventory difference can be calculated by element and by isotope for each nuclear material:

ID = AQ - DQ = AQ - (IQ+INC - DEC), (18)

where:

AQ is the actual available quantity of nuclear materials in the material balance area determined in the course of this physical inventory;

DQ is the documented quantity of nuclear materials in the material balance area as of the beginning of the physical inventory taking (end of this MBP);

INC is the documented increase of nuclear material quantity in the material balance area for this inter-balance period due to all supplies, production, etc;

DEC is the documented decrease of nuclear material quantity in the material balance area for this inter-balance period due to all shipments from MBA, nuclear transformations, losses. etc;

IQ is the documented quantity of nuclear materials in material balance area as of beginning of this inter-balance period.

The ID equation can be represented as sum of nuclear material (element or isotope).

$ID= \left(\sum\_{i=1}^{N1}m\_{i}+ \sum\_{i=1}^{N2}m\_{i}+ \sum\_{i=1}^{N3}m\_{i}- \sum\_{i=1}^{N4}m\_{i}\right)$, (19)

where:

N1 is number of inventory items in IQ;

N2 is number of inventory items in INC;

N3 is number of inventory items in DEC;

N4 is number of inventory items in AQ;

mi - weight of element when determining ID by the element (mass fraction C):

mi = mHi Ci, (20)

where:

mHi - net weight;

mi - weight of isotope when determining ID by the isotope (mass fraction Ci):

mi = mHi Ci Cii. (21)

33. To assess ID error, the measurement error transfer method can be used.

34. Due to the fact that measurement of nuclear material quantity is performed indirectly, as a rule, it is advised to represent the ID variance, in general, by square diagonal matrix of variances and covariances of measurement results, which contains 4 rows and columns based on number of components of the material balance equation, and has form of:

 │ 2 │

 │sigma rho sigma sigma rho sigma sigma rho sigma sigma │

 │ 1 12 1 2 13 1 3 14 1 4│

 │ │

 │ 2 │

 │rho sigma sigma sigma rho sigma sigma rho sigma sigma │

 2 │ 21 1 2 2 23 2 3 24 2 4│

sigma = │ │ (22)

 ir │ 2 │

 │rho sigma sigma rho sigma sigma sigma rho sigma sigma │

 │ 31 1 3 32 2 3 3 34 3 4│

 │ │

 │ 2 │

 │rho sigma sigma rho sigma sigma rho sigma sigma sigma │

 │ 41 1 4 42 2 4 43 3 4 4 │

Taking into account (22), the expression for calculation of inventory difference error can be presented as follows:

$sigma\_{ID}= \sqrt{\sum\_{i=1}^{4}sigma\_{i} \sum\_{j=1}^{4}po\_{ij} sigma\_{j}}$, (23)

where:

i - row index of variance and covariance matrix;

j - column index of variance and covariance matrix;

sigma i - error of the measurement results of nuclear material quantity in the component of balance equation with index i;

sigma j - error of the measurement results of nuclear material quantity in the component of balance equation with index j;

rhoij - coefficient of correlation between components of the balance equation with indices i and j.

The structure of error of the NM measurement results is considered in section VI of this Provision. This section also provides analytical expressions for calculating variances and covariances. In such calculations the data contained in documentation for measuring instruments in methods of measurement can be used.

The reason for correlation between components of the balance equation can be use of the same measuring equipment, when performing NM inventory measurements.

35. To calculate the MSD values of each components of the balance equation, it is advised to split all measured inventory items into several (K) independent (uncorrelated) strata. In general, the variance of NM measurement results in inventory item of the k-th sigma$\genfrac{}{}{0pt}{}{2}{k}$ stratum, with due regard to possible correlations of determining of nuclear material parameters in the inventory item that make up this stratum, can be calculated by formula:

$sigma\genfrac{}{}{0pt}{}{2}{k}= \sum\_{l=1}^{N\_{k}}sigma\genfrac{}{}{0pt}{}{2}{1} (M\_{kl})+ 2\sum\_{l=1}^{N\_{k}-1}\sum\_{m=l+1}^{N\_{k}}rho \left(M\_{kl}, M\_{km}\right)sigma\_{l}\left(M\_{kl}\right)sigma\_{m}(M\_{km})$*,* (24)

where:

$sigma\genfrac{}{}{0pt}{}{2}{1} (M\_{kl})$ - variance of calculation of nuclear material weight of the l-th inventory item included in the k-th

stratum containing Nk inventory item, taking into account the random and systematic error components of calculation of the NM weight in the l-th inventory item - Mkl;

the indices l and m correspond to the sequence numbers of inventory item included in the k-th stratum;

rho (Mkl, Mkm) - coefficient of correlation between NM weight values in the l-th inventory item - Mkl and in the m-th inventory item - Mkm included in the k-th stratum.

The value of correlation coefficients can vary from -1 to +1.

For independent values the correlation coefficient between them equals zero.

Calculation of the correlation coefficient value during analysis of measurement results is a quite time-consuming task. Therefore, in most cases, to simplify the processing of measurement results and calculation of sigmaID value, it is advised to ignore possible correlations between components of the balance equation and between results of measurements of NM parameters in inventory item strata as well. The values of sigmaID and sigma$\frac{2}{k}$ are determined according to following expressions:

$sigma\_{ID}= \sqrt{\sum\_{i=1}^{4}sigma\genfrac{}{}{0pt}{}{2}{i}}$, (25)

$sigma\genfrac{}{}{0pt}{}{2}{k}= \sum\_{l=1}^{N\_{k}}sigma\genfrac{}{}{0pt}{}{2}{1}(M\_{kl})$. (26)

This will eventually lead to certain decrease in the found value of sigmaID and, consequently, increase in risk of error of the first kind (registration of deviation in tracing and control, when the deviation is actually missing). However, it should be assumed that with correct organization of tracing and control of nuclear material in the material balance area, this phenomenon will occur quite rarely, which makes it acceptable to assume that there are no correlations between corresponding values, when calculating sigmaID, sigma$\frac{2}{k}$ in practical tasks.

If there are still good reasons to consider correlations, the organization would develop necessary techniques to determine appropriate correlation coefficients.

36. To determine the variance of nuclear material weight in inventory item in the organization, the organization can develop techniques for calculation of nuclear material weight and its error, taking into account the material balance area equipment with technical facilities for carrying out inventory measurements, MM and process's features.

37. When calculating the error of weight measurement, it is advised to consider error sources such as: statistical error of inventory item selection, bulk measurement error, material sampling error, error of material chemical analysis and others, depending on process's features.

38. For those inventory item strata with nuclear material in the material balance area that were not subjected to any transformations during inter-balance period, when calculating ID and sigmaID, it is advised to use inventory data obtained earlier for the inventory items being part of them. However, such data do not have any effect on sigmaID value.

39. To calculate sigmaID value and assess results for presence of possible deviations in tracing and control of nuclear materials, the procedure set out In the standard of JSC TVEL Corporation "System for recording and control of nuclear materials" can be used, according to the procedure:

determination of MSD of systematic (sigmaS) and random (SigmaR) error components of all nuclear material measurement techniques in the material balance area;

preparation of data for recording the NM movement for calculating ID and sigmaID and selection of NM groups (strata);

calculating ID by element and isotope for each nuclear material in the material balance area;

exclusion of "paired inventory data" from sigmaIDcalculation, which should not affect sigmaID value;

sigma$\genfrac{}{}{0pt}{}{2}{ID}$ calculation (ID variance);

checking according to the criteria for detecting deviations in NM tracing and control procedure.

40. When determining characteristics of measurement errors (MSD of systematic and random components) the following is advised:

establish random and systematic components of the characteristics of NM measurement errors in the material balance area based on method of NM measurement;

to calculate sigmaID value, calculate the mean square deviation value for each random and systematic error component of all methods of NM measuring in the material balance area. If error of method of measurement is regulated in absolute form - DELTA, then follow the additive error model. If error of method of measurement is regulated in relative form - delta, then follow the multiplicative model. The description of additive and multiplicative models of measurement error and relationship with the absolute and relative measurement errors are given in Appendix 6 to this Provision.

The transition from the interval error characteristics to MSD (sigma) at P = 0.95 can be done as follows:

$sigma\_{R}= \frac{DELTA\_{R}}{1,96}$ - Mean square deviation of random component for theabsolute error (additive model);

$sigma\_{R}= \frac{DELTA\_{R}}{1,96}$ - Mean square deviation of random component for the relative error (multiplicative model);

where:

DELTA R, deltaR - interval characteristics of random component of the measurement error at P = 0.95;

$sigma\_{S}= \frac{THETA (DELTA)\_{S}}{1,96}$ - Mean square deviation of systematic component for the absolute error (additive model);

$sigma\_{S}= \frac{THETA (DELTA)\_{S}}{1,96}$ - Mean square deviation of systematic component for the relative error (multiplicative model);

where: THETA(DELTA)S, THETA(delta)S - interval characteristics of systematic component of the measurement error at P = 0.95.

For weighing procedures, at P = 1 it is advised to use certified error values for calculations.

After all the characteristics of measurement errors are determined, it is advised to develop final report (reference) that is compliant with the company's documents.

Example of recommended presentation form for the final results related to definition of MSD components of measurement errors is presented in Appendix 7 to this Provision.

41. When preparing data for recording the movement of nuclear material for calculating inventory difference and sigmaID, when selecting NM groups (strata) YAM the following is advised:

record of all supplies and other increase of nuclear material in the material balance areas, dispatches and other decrease of nuclear material (for example, waste generation) from the material balance areas, results of previous and current physical inventory in the report documents according to company's documents for NM recording and control. Example of registration the NM movement for material balance area for fuel element production is presented in table P8. 1 of Appendix 8 to this Provision;

define nuclear material groups (strata) Each group (stratum) is characterized by set of the same methods of measurements (for example: weighing on the same scales, determining the mass fraction of the element for the same method of measurement, determining isotope mass fraction for the same method of measurement) and respective error characteristics (see example in table P8. 1 of Appendix 8 to this Provision).

Each group (stratum) includes information on NM movement of in the material balance area for inter-balance period. Example of selection of groups (strata) for material balance area of fuel element production is presented in Tables P8.2, P8.3 and P8.4 of Appendix 8 to this Provision.

42. For calculation of inventory difference use formula (20). The calculations are performed for each nuclear material (element and isotope).

43. To exclude the "paired inventory records" from calculation of Sigma ID the following is advised:

when analyzing the NM movement record table (see example - Table P8.1, Appendix 8 to this Provision), select inventory records that correspond to the same inventory item (with the same identification number), which remained unchanged and which the works with it for inter-balance period were not performed, and reliability of inventory data for this inventory item was confirmed by proper state of the access-control means involved since time, when the data was determined till the moment it was used;

when for each of such inventory unit exist two inventory records that correspond to different terms of the ID equation with opposite signs, then such "paired inventory records" shall be removed from sigmaID calculation to prevent unjustified increase in sigmaID. For example, following types of "paired inventory records" are possible:

Members of equation of inventory difference between INC and AQ.

Received to material balance area inventory items (INC) registered according to the sender's data, then during physical inventory, the same inventory items were entered into inventory list (AQ).

Members of equation of inventory difference between INC and DEC.

Inventory items received in material balance area (INC), which were then sent out from the material balance area (DEC).

Members of equation of inventory difference between IQ and DEC.

Part of inventory items from previous physical inventory (IQ) were sent from material balance area (DEC).

Members of equation of inventory difference between IQ and AQ.

Part of inventory items from previous physical inventory (IQ) was included in the list of inventory items of the current inventory (AQ).

Example of exclusion of the paired records is given in Appendix 9 to this Provision.

44. Calculation of SigmaID is advised to be performed both with averaging and without averaging over the stream.

The stream averaging approach assumes that, on average, all the inventory weights in NM batch are approximately the same and weight of one inventory item equals to average weight of inventory item the batch. When using this approach, the resulting sigmaID value usually determines whether it is appropriate to apply more strict criteria for detecting deviations than in case of calculating the value without stream averaging.

The approach fo calculation Sigma$\frac{2}{ID}$ without averaging over the stream consists in using individual inventory weights of nuclear material for each inventory item. To implement this approach, it is advised to develop special software.

The general sequence of calculating sigmaID and testing the hypothesis on deviation absence for these two methods remains the same.

When making decision about using particular method, it is advised to make an assessment of the value, by which the calculated sigma$\frac{2}{ID}$ decreases with stream averaging (Appendix 10 to this Provision).

45. When calculating Sigma$\frac{2}{ID}$ (with stream averaging), analytical expressions for relative measurement errors are advised to be used as the most commonly used expressions in practice, when calculating the ID variance (multiplicative model).

In this case, the contribution of the systematic error component for one of the measurement methods (weighing, volume measurement, and other methods of measurements) to the ID variance for the j-th group (stratum) can be determined using following expression:

$V\_{Sj}=M\frac{2}{NMSj} sigma\frac{2}{Sj}$*,* (27)

where SigmaSj - mean square deviation of systematic error component of one of the measurement methods,

$М\_{NMSj}= \sum\_{i belongs Strj}^{}sgn \left(m\_{i}\right)m\_{NMi}$, (28)

sgn (mi) = +1 for mi from IQ, INC - terms of ID equation,

sgn(mi) = -1 for mi from AQ, DEC - terms of ID equation. This approach is valid, for example, when weighing the entire nuclear material that makes up the terms of ID equation with the same scales without recalibration.

When weighing and determining the element (isotope) mass fraction , the following approach can be used.

When calculating the inventory difference by element:

mNMi = (mбi - mti) Ci, (29)

where:

mбi - gross weight;

mti - tare weight;

Ci - mass fraction of the element.

For calculating ID by isotope:

mNMi = (mбi - mti) Ci Cii, (30)

where Sii - isotope mass fraction.

When calculating contribution of the systematic error component to Sigma$\frac{2}{ID}$, the total sum of NM weights in the group (stratum) is determined, with consideration of their sign in ID equation. Due to the fact that results of the sum of weights for method with stream averaging and method without averaging are the same, these two approaches provide equivalent results. The procedure for calculating the total mass of MNMj of the j-th group (stratum) to determine contribution of systematic and random error components to Sigma$\frac{2}{ID}$ is provided below.

The contribution of random error component for one of the measurement methods (weighing, volume measurement, measurement method of destructive and non-destructive testing methods) to sigma$\frac{2}{ID}$ for the j-th group (stratum) can be calculated using following expression:

$V\_{Rj}= \frac{(M\_{NMRj})^{2}}{N\_{j}} sigma\frac{2}{R\_{j}}$*,* (31)

where:

MNMRj - total mass of the element (isotope) of the j-th group (stratum);

Nj - number of measurements in the j-th group (stratum);

SigmaRj - random error component of one of the measurement methods of the j-th group (stratum).

Total mass of the element (isotope) of the j-th group (strata):

 R

 M = SUM M , (32)

 NMRj i=1 NMi

where:

R - number of batches in the j-th group (stratum);

MNMi - total mass of the element (isotope) in the i-th batch.

Total number of weighings (volume measurements, etc.) in the j-th group (stratum):

 R

 N = SUM N , (33)

 j i=1 i

where:

R - number of batches in the j-th group (stratum);

Ni - number of containers (volume measurements) in the i-th batch.

Total number of analyses by measurement method using one of the destructive (non-destructive) testing methods in the j-th group (stratum):

 N = R х p , (34)

 j i

where:

R - number of batches of nuclear material, where the mass fraction of the element (isotope) is determined by measurement method for the j-th group (stratum);

pi - number of samples per batch.

Analytical expressions for calculating sigma$\frac{2}{ID}$ without stream averaging are considered in Appendix 11 to this Provision.

When calculating the total mass of MNMj in the j-th group (stratum) to determine contribution of systematic and random error components in sigma$\frac{2}{ID}$ (with stream averaging) to its beginning, it is advised to perform exclusion of "paired inventory records".

To calculate contribution of systematic error components to sigma$\frac{2}{ID}$, it is advised to sum masses of the element (isotope) considering sign of ID equation for the case when the same measurement tools are used to weigh the entire nuclear material that makes up terms of ID equation without recalibration. In example of excluding the paired records presented in Appendix 9 to this Provision, the sums of masses of the element (isotope) are summed up and recorded in the line MNMS (syst) by the element and isotope:

MNMS (syst) = Sum (IQ) + Sum (INC) - Sum (AQ) - Sum (DEC). (35)

To calculate contribution of systematic error components to sigma$\frac{2}{ID}$, it is advised to sum masses of the element (isotope) without consideration of sign of ID equation for the case, when the same measurement tools are used to weigh the entire nuclear material that makes up terms of ID equation without recalibration. In example of excluding the paired records presented in Appendix 9 to this Provision, the sums of masses of the element (isotope) are summed up and recorded in the line MNMS (rand) by the element and isotope:

MNMR (rand) = Sum (IQ) + Sum (INC) + Sum (AQ) + Sum (DEC). (36)

Example of calculating the total mass of the material is provided in Appendix 12 to this Provision.

When calculating Sigma$\frac{2}{ID}$, the following information can be used:

- error characteristics (sigmaR and sigmaS);:

- number of measurements (number of containers at single weighing);

- number of samples taken from nuclear material batch to determine the mass fraction value of the element and isotope.

Appendix 13 to this Provision provides example of calculating the contribution of systematic and random error components of the j-th group (stratum) to sigma$\frac{2}{ID}$.

To calculate contribution of systematic error components to Sigma$\frac{2}{ID}$ for the measurement methods performed in the j-th group (stratum), following expression (27) can be used. The total contribution of systematic error components of the j-th group (stratum) to sigma$\frac{2}{ID}$ shall be determined.

For example:

- VSMj - contribution of systematic component of weighing error of the j-th group (stratum) to sigma$\frac{2}{ID}$;

- VSAj - contribution of systematic error component of the destructive analysis (by element and by isotope) of the j-th group (stratum) to Sigma$\frac{2}{ID}$.

The total contribution to sigma$\frac{2}{ID}$ of systematic error components is defined as sum of individual contributions (weighing and analysis):

 V = V + V . (37)

 S SUMj SMj SAj

To calculate contribution of systematic error components to sigma$\frac{2}{ID}$ for the measurement methods performed in the j-th group (stratum), following expression (28) can be used. The total contribution of random error components of the j-th group (stratum) to sigma$\frac{2}{ID}$ shall be determined.

For example:

- VRM - contribution of random component of the weighing error of the j-th group (stratum) to sigma$\frac{2}{ID}$;

- VRA - contribution of random component of the destructive analysis error(by element and by isotope) of the j-th group (stratum) to sigma$\frac{2}{ID}$.

The total contribution to sigma$\frac{2}{ID}$ of random error components is defined as sum of individual contributions (weighing and analysis):

 V = V + V . (38)

 R SUMj RMj RAj

The contributions to Sigma$\frac{2}{ID}$ of systematic error components of all groups (strata) are summed:

 K

 V = SUM V , (39)

 S SUM j=1 S SUMj

where:

VS SUMj - contribution of systematic error components to Sigma$\frac{2}{ID}$ of the j-th group (stratum);

K - number of groups (strata).

The contributions to sigma$\frac{2}{ID}$ of random error components of all groups (strata) are summed:

 K

 V = SUM V , (40)

 R SUM j=1 R SUMj

where VR SUMj - contribution of random error components to sigma$\frac{2}{ID}$ of the j-th group (stratum).

It is advised to make a comparative analysis of contribution of each of the error components to Sigma$\frac{2}{ID}$.

The total sigma$\frac{2}{ID}$ consists of systematic and random components of measurement errors of all groups (strata):

 2

 sigma = V + V . (41)

 ID S SUM R SUM

Mean square deviation of inventory difference:

 \_\_\_\_\_\_

 / 2

 sigma = \/sigma . (42)

 ID ID

46. As a criterion for detecting deviations in the record and control of nuclear material for inter-balance period, it is advised to use compliance with at least one of the inequalities in the following system

 ┐

 |ID| > 3 sigma │

 ID

 │

 |ID| > G │

 } (43)

 |ID| > 0,02 - for industrial nuclear installations

 │

 |ID| > 0,03 - for nuclear research │

 facilities and factory laboratories

 ┘

The numerical values in the third and fourth inequalities of the system (43) determine critical fractions of the total number of a given nuclear material converted and subjected to the inventory measurements in given inter-balance period or in process of physical inventory..

The second inequality of the system can be checked only for nuclear materials, for which G thresholds are set.

The check of compliance with the third and fourth inequalities can be performed separately for each nuclear material in material balance area by element and by isotope.

Appendix 1
 to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

EXAMPLE OF COMPARING TWO SPECTRUM PROCESSING ALGORITHMS

When developing standard samples of plutonium content for inspection measurements, the samples were measured using U-Pu InSpector inspection gamma-ray spectrometric station with a low-energy germanium detector (LEGe) with active area of 5 sq. cm. To obtain data on isotopic composition the MGA software was used. The measurement results were processed to compare them with the data obtained from destructive analysis of material samples and for comparison the two algorithms provided by the station's software: with or without Declaration of Pu-242 content prior to sample measurement.

There were measured 11 sets of standard samples, each of the set included four samples of different isotopic composition. The information on composition was obtained using destructive methods for analyzing material samples prior to production of samples and presented prior to inspection measurements. For example, the percentage of Pu-239 in samples of composition No. 1 was equal to 94.176 +/- 0.020, for composition No. 2 it was equal 90.034 +/- 0.026, for composition No. 3: 85.06 +/- 0.06, and for the composition No. 4: 79.24 +/- 0.02. For each of 44 samples, eight parallel measurements of isotopic composition were performed with spectrum processing using an algorithm that provides declaration of Pu-242 content before measuring the sample. To process each ninth spectrum, an algorithm was used that does not require declaring the Pu-242 content. Resolution at 122 keV for the measurements was (590 - 600) eV (FWHM). The gathering time for each spectrum was 1000 s. By changing distance between the detector and the sample and selecting thickness of the filter to reduce intensity of low-energy photons, the "dead" time for samples of different contents was adjusted in the range of (18-22)%. In such event, the total number of counts during gathering of each spectrum was (6 - 8) x 106, and in energy regions at (59.54; 129.29; 208.00) keV it was minimum 105 for comparable values. The displacement of centroid lines corresponding to the listed energies was found within five channels during the measurement process. There was no need to correct the gain offset during the measurement period. The multiplier corresponding to gain in the calibration equation was approximately 0.075 keV.

At stage of preliminary analysis, the following checks were performed for each sample for 8 parallel determinations of each isotope content using the existing methods:

consent of experimental distribution with the theoretical one;

stochastic independence of results of parallel definitions;

minor discrepancies in results of parallel definitions;

insignificant differences in variances according to the Cochran test;

homogeneity

To check the consent of experimental distribution with the normal law, the value of criterion for small sample volumes was compared with the critical value for significance level of 0.05. In the course of such investigation, the discrepancy between these parallel definitions and the normal law for significance level of 0.05 was not found.

The stochastic independence of parallel definitions results was checked using criterion of ratio of consecutive difference squares. In the course of such investigation, no cases of violation of the inequality for significance level of 0.05 were found.

Checking insignificance of the discrepancy between results (xi, i = 1,...,n) for each series of n = 8 parallel definitions for a confidence probability of 0.95 did not reveal bad values in the experimental data. Verification of variances for each isotope of samples of each content did not reveal significant differences in variances.

When investigating homogeneity of standard samples of plutonium content for each content for each isotope, the mean square deviation that characterizes heterogeneity of the material by content of each isotope in the samples of each plutonium content, Sigman was calculated.

After no significant difference was found between parallel determinations of the content of each analyzed isotope in standard samples of the same plutonium content, the data for such samples was combined.

When checking consent of experimental distributions of combined data with the normal law (number of measurements exceeds fifty) according to omega2 criterion, no evidence of non-compliance for the significance level of 0.05 of experimental distribution with the normal law was found.

At the next stage, the tables for each isotope were made, results of the sample measuring of the same plutonium content were recorded in each of the four columns. To analyze the tables, the usual integrated statistical package was used, which allows performing standard statistical calculations without digging into into the theory. For the table for analyzed isotope, arithmetic means and measurement variances of four standard samples of plutonium content were calculated, the variances were compared according to the Fischer variance ratio test, and type of measurement error model was determined. Then the random and systematic components of measurement errors were determined using one-factor analysis of variance involving following algorithm. If in $\sum\_{i=1}^{k}\sum\_{j=1}^{n}(\hat{y\_{ij}}- \overline{y})^{2}=n\sum\_{i=1}^{k}(\overline{y\_{i}}- \overline{y})^{2}+ \sum\_{i=1}^{k}\sum\_{j=1}^{n}(\hat{y\_{ij}}- \overline{y})^{2}$ the following designations are entered: $V\_{1}=n \sum\_{i=1}^{k}(\overline{y\_{i}}- \overline{y})^{2}$, $V\_{2}= \sum\_{i=1}^{k}\sum\_{j=1}^{n}(\hat{y\_{ij}}- \overline{y\_{1}})^{2}$and assume that $\overline{y\_{i}} ∼= N(0, sigma\genfrac{}{}{0pt}{}{2}{S}+sigma\genfrac{}{}{0pt}{}{2}{R} / n)$ is distributed according to the normal law (index "R" refers to the random component of measurement errors, and "S" - to the systematic component), then we get:

 2

 V sigma

 1 2 R

 --------- = sigma + ------,

 n (k - 1) S n

 V V V

 /\ 2 1 2 /\ 2 2

 sigma = --------- - -----------, sigma = ---------.

 S n (k - 1) k (n - 1) n R k (n - 1)

Here k = 4 - number of standard samples in the set; n = 88 - number of measurements of samples of the same plutonium content; $\hat{y\_{ij}}$ - for the i-th sample of the j-th content, difference between value measured using U-Pu InSpector and value obtained in the analytical laboratory;$\overline{y}= \left(\sum\_{i=1}^{k}\sum\_{j=1}^{n}\hat{y\_{ij}}\right) / kn$;$y\_{i}= \left(\sum\_{j=1}^{n}\hat{y\_{ij}}\right) / n$.

Similarly, the measurement results were processed without Declaration of Pu-242 content. Only in this case 11 x 4 tables were analyzed.

The results of processing are shown in the tables below (in each cell of the table data for samples of 1, 2, 3 and 4 composition are sequentially recorded ).

Table P1.1

RESULTS OF MEASUREMENT ANALYSIS WITH DECLARATION OF PU-242 CONTENT

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| IsotopeCharacteristic | Pu-238 | Pu-239 | Pu-240 | Pu-241 | Am-241 |
| The mean square deviation that characterizes inhomogeneity of material in standard Sigman sample for each isotope for samples of the same plutonium content | 1.44 х 10-41.65 х 10-42.37 х 10-41.76 х 10-4 | 1.49 х 10-20.93 х 10-22.13 х 10-21.56 х 10-2 | 1.48 х 10-20.94 х 10-22.00 х 10-21.47 х 10-2 | 1.59 х 10-42.49 х 10-41.98 х 10-43.66 х 10-4 | 1.05 х 10-40.96 х 10-41.57 х 10-41.43 х 10-4 |
| Random component of measurement errors for each isotope for samples of the same plutonium content | 3.81 х 10-45.36 х 10-46.21 х 10-46.00 х 10-4 | 3.33 х 10-23.61 х 10-25.29 х 10-25.35 х 10-2 | 3.33 х 10-23.58 х 10-25.22 х 10-25.27 х 10-2 | 3.94 х 10-46.63 х 10-48.24 х 10-49.15 х 10-4 | 2.9 х 10-43.4 х 10-43.3 х 10-43.3 х 10-4 |
| Systematic component of measurement errors for each isotope for samples of the same plutonium content | 4.11 х 10-42.18 х 10-42.84 х 10-42.32 х 10-4 | 3.94 х 10-32.75 х 10-32.61 х 10-34.23 х 10-3 | 3.85 х 10-32.75 х 10-32.56 х 10-34.22 х 10-3 | 2.50 х 10-41.39 х 10-41.62 х 10-42.03 х 10-4 | 3.05 х 10-51.80 х 10-52.60 х 10-52.05 х 10-5 |
| Confidence limits (interval) of random error of the Epsilon measurement result for each isotope for samples of the same plutonium content | 0.83 х 10-41.17 х 10-41,35 х 10-41.31 х 10-4 | 0.72 х 10-20.77 х 10-21.14 х 10-21.15 х 10-2 | 0.72 х 10-20.77 х 10-21.12 х 10-21.13 х 10-2 | 0.83 х 10-41.40 х 10-41,74 х 10-41.93 х 10-4 | 6.2 х 10-57.2 х 10-57.1 х 10-57.0 х 10-5 |
| Upper bound of the relative mean square deviation for results of BSR determinations for each isotope for samples of the same plutonium content | 1.01 х 10-20.32 х 10-20.29 х 10-20.32 х 10-2 | 3.56 х 10-44.03 х 10-46.27 х 10-46.81 х 10-4 | 0.59 х 10-20.38 х 10-20.37 х 10-20.26 х 10-2 | 0.41 х 10-20.32 х 10-20.29 х 10-20.28 х 10-2 | 3.76 х 10-25.56 х 10-28.51 х 10-27.08 х 10-2 |
| Confidence limits of error of DELTA assessment result Aco for each isotope for samples of the same plutonium content | 4.19 х 10-42.47 х 10-43.14 х 10-42.66 х 10-4 | 8.21 х 10-38.18 х 10-311.70 х 10-312.25 х 10-3 | 8.17 х 10-38.18 х 10-311.49 х 10-312.06 х 10-3 | 2.63 х 10-41.97 х 10-42.38 х 10-42.80 х 10-4 | 6.91 х 10-57.42 х 10-57.56 х 10-57.29 х 10-5 |
| Confidence limits of error with consideration of heterogeneity of DELTA samplesco for each isotope for samples of the same plutonium content | 5.05 х 10-44.07 х 10-45.61 х 10-44.36 х 10-4 | 3.03 х 10-22.00 х 10-24.34 х 10-23.29 х 10-2 | 3.01 х 10-22.02 х 10-24.08 х 10-23.12 х 10-2 | 4.08 х 10-45.26 х 10-44.55 х 10-47.70 х 10-4 | 2.17 х 10-42.02 х 10-43.17 х 10-42.90 х 10-4 |

Table P1.2

RESULTS OF MEASUREMENT ANALYSIS WITHOUT DECLARATION OF PU-242 CONTENT

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| IsotopeCharacteristic | Pu-238 | Pu-239 | Pu-240 | Pu-241 | Pu-242 | Am-241 |
| The mean square deviation that characterizes inhomogeneity of material in standard Sigman sample for each isotope for samples of the same plutonium content | 3.30 х 10-46.17 х 10-45.05 х 10-43.81 х 10-4 | 2.17 х 10-22.19 х 10-26.20 х 10-25.09 х 10-2 | 2.16 х 10-22.15х 10-26.12 х 10-24.78 х 10-2 | 3.82 х 10-46.58 х 10-45.43 х 10-410.13 х 10-4 | 0.99 х 10-41.04 х 10-47.70 х 10-432.62 х 10-4 | 2.10 х 10-43.52 х 10-43.33 х 10-43.85 х 10-4 |
| Random component of measurement errors for each isotope for samples of the same plutonium content | 3.23 х 10-46.04 х 10-44.95 х 10-43.73 х 10-4 | 2.10 х 10-22.12 х 10-26.00 х 10-24.92 х 10-2 | 2.16 х 10-22.15х 10-26.11 х 10-24.77 х 10-2 | 3.87 х 10-46.67 х 10-45.50 х 10-410.27 х 10-4 | 0.44 х 10-40.46 х 10-43.41 х 10-414.44 х 10-4 | 5.26 х 10-38.81 х 10-38.34 х 10-39.63 х 10-3 |
| Systematic component of measurement errors for each isotope for samples of the same plutonium content | 1.07 х 10-30.57 х 10-30.74 х 10-30,60 х 10-3 | 2.48 х 10-21.70 х 10-21.61 х 10-22.61 х 10-2 | 1.08 х 10-20.77 х 10-20.72 х 10-21.18 х 10-2 | 6.76 х 10-43.77 х 10-44.38 х 10-45.50 х 10-4 | 2.29 х 10-21.22 х 10-21.63 х 10-2154 х 10-2 | 6.36 х 10-33.61 х 10-33.73 х 10-33.68 х 10-3 |
| Confidence limits (interval) of random error of the Epsilon measurement result for each isotope for samples of the same plutonium content | 2.33 х 10-44.34 х 10-43.56 х 10-42.68 х 10-4 | 1.53 х 10-2154 х 10-24.37 х 10-23.59 х 10-2 | 1.52 х 10-21.57 х 10-24.31 х 10-23.37 х 10-2 | 2.69 х 10-44.64 х 10-43.83 х 10-47.14 х 10-4 | 0.70 х 10-40.74 х 10-45.43 х 10-422.99 х 10-4 | 1.48 х 10-42.48 х 10-42.35 х 10-42.71 х 10-4 |
| Upper bound of the relative mean square deviation for results of BSR determinations for each isotope for samples of the same plutonium content | 1.00 х 10-20.42 х 10-20.27 х 10-20.23 х 10-2 | 2.69 х 10-42.83 х 10-48.52 х 10-47.55 х 10-4 | 4.48 х 10-32.66 х 10-35.02 х 10-32.79 х 10-3 | 4.62 х 10-33.77 х 10-32.29 х 10-33.69 х 10-3 | 6.59 х 10-33.25 х 10-33.18 х 10-35.87 х 10-3 | 3.14 х 10-26.71 х 10-29.97 х 10-29.47 х 10-2 |
| Confidence limits of error of DELTA assessment result Aco for each isotope for samples of the same plutonium content | 1.10 х 10-30.72 х 10-30.82 х 10-30.66 х 10-3 | 2.91 х 10-22.29 х 10-24.66 х 10-24.44 х 10-2 | 1.86 х 10-21.75 х 10-24.37 х 10-23.57 х 10-2 | 7.28 х 10-45.98 х 10-45.82 х 10-49.01 х 10-4 | 2.29 х 10-21.22 х 10-21.63 х 10-21.56 х 10-2 | 6.36 х 10-33.62 х 10-33.74 х 10-33.69 х 10-3 |
| Confidence limits of error with consideration of heterogeneity of DELTA samplesco for each isotope for samples of the same plutonium content | 1.27 х 10-31.41 х 10-31.29 х 10-31.00 х 10-3 | 5.16 х 10-24.87 х 10-213.01 х 10-210.92 х 10-2 | 4.63 х 10-24.56 х 10-212.77 х 10-210.03 х 10-2 | 1.04 х 10-31.42 х 10-31.21 х 10-32.18 х 10-3 | 2.29 х 10-21.22 х 10-21.64 х 10-21.68 х 10-2 | 6.38 х 10-33.68 х 10-33.79 х 10-33.77 х 10-3 |

The calculated values of the calibration characteristics parameters y = ax + b, where y - measured value of Pu isotope content, and x - value obtained in an analytical laboratory, are shown in the tables below.

Table P1.3

CHARACTERISTICS OF STATISTICAL MODELS FOR MEASUREMENTS WITH DECLARATION OF PU-242 CONTENT

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| IsotopeCharacteristic | Pu-238 | Pu-239 | Pu-240 | Pu-241 |
| Parameter a (parameter standard error) | 1.023(3.13 x 10-3) | 1.007(0.44 x 10-3) | 1.007(0.45 x 10-3) | 0.992(1.58 x 10-3) |
| Parameter b (parameter standard error) | -4.97 х 10-3(5.37 x 10-4) | -6.99 х 10-1(3.85 x 10-2) | -2.86 х 10-2(6.08 x 10-3) | -8.72 х 10-3(3.95 x 10-4) |
| Sum of squares SSD determined by regression | 1.72 | 1.11 х 104 | 1.03 х 104 | 2.56 |
| Residual sum of squares SSR | 5.65 х 10-3 | 0.746 | 0.730 | 2.26 х 10-3 |
| Correlation coefficient r | 0.998 | 0.999 | 0.999 | 0.999 |

Table P1.4

CHARACTERISTICS OF STATISTICAL MODELS FOR MEASUREMENTS WITHOUT DECLARATION OF PU-242 CONTENT

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| IsotopeCharacteristic | Pu-238 | Pu-239 | Pu-240 | Pu-241 | Pu-242 |
| Parameter a (parameter standard error) | 1.021(8.14 x 10-3) | 1.034(2.79 x 10-3) | 1.002(1.26 x 10-3) | 0.987(4.25 x 10-3) | 2.259(0.338) |
| Parameter b (parameter standard error) | -4.98 х 10-3(1.40 x 10-3) | -3.032(0.244) | 0.017(0.017) | -7.76 х 10-3(1.07 x 10-3) | -0.152(0.066) |
| Sum of squares SSD determined by regression | 0.214 | 1465 | 1278 | 0.316 | 1.473 |
| Residual sum of squares SSR | 5,72 х 10-4 | 0.449 | 0.085 | 2,46 х 10-4 | 1.386 |
| Correlation coefficient r | 0.999 | 0.999 | 0.999 | 0.999 | 0.718 |

The considered example demonstrates how error characteristics of the results for determining the isotope content in samples can be calculated using experimental data by processing them; estimate the parameters of calibration characteristics for various algorithms for processing spectra: when Pu-242 content is declared before performing the measurement and for the case without such declaration.

When comparing the error characteristics of the results of determining the isotope content in standard samples, deterioration of most of these characteristics is noted, when algorithm without declaring the Pu-242 content is used. This is most noticeable for Am-241.

For all Pu isotopes, regression models for significance value of 0.05 were found to be adequate to the experimental data for the case with declaration of Pu-242 content and without it. For Pu-242, the quality of the regression model was lower than for other Pu isotopes. The inclination angle of the calibration characteristic for Pu-242 differs significantly from 45°, and for Pu-239 this angle is significantly biased down for the case of determination the Pu-242 content in samples with software tools.

When analyzing the measurement error components, there is a tendency to decrease Pu-238 systematic component with increase in the content of this isotope in case of using both spectrum processing algorithms. The random component for Pu-239 decreases, and for other Pu isotopes it increases with increasing content of the corresponding isotope in case of using the spectrum processing algorithm with declaration of Pu-242 content. No other trends to increase (decrease) the error components for significance level of 0.05 were found.

The error characteristics of results that show the content of Pu isotopes of this Appendix and identified features of the spectrum processing algorithms proposed by software developers shall be considered to record and control the nuclear material.

Appendix 2
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

PROCEDURE FOR CALCULATING THE SAMPLE VOLUME

The sample volume is set by two values: the threshold of nuclear material quantity in mass units (G) and probability of detecting a shortage/excess of the threshold of nuclear material quantity in non-relative units (P). These values are determined in accordance with The Fundamental Rules for Accounting and Control of Nuclear Materials (NP-030-11). The results of checking the integrity of unibody inventory items and ACM state are documented for documentary confirmation that unauthorized access is not available.

Before calculating the sample volume, the G / x ratio shall be determined.

The sample volume (n) is calculated as follows:

when the value of G / x <= N, the sample volume (n) is calculated by following:

n = [N (1 - (1 - P)1/[G/x])]+, (P 2.1)

where:

N - number of inventory items in stratum;

x - average weight of nuclear material in one inventory item;

[ ]+ - in the formula means rounding to the nearest larger integer;

when the value of G / x > N is subject to verification of all inventory units.

Calculation of the sample value of the seals to be checked in accordance with item 25 of this Provision shall be carried out using following equation n = [N (1 - (1 - 0,95)1/(0,05 N))]+; where N - number of installed seals in the material balance area. Here, [ ]+ also means rounding to the nearest larger integer. The results of this calculation for several n values are shown below.

|  |  |
| --- | --- |
| Seals total, pieces | Number of seals checked for their operability and state |
| 10 | 10 |
| 20 | 19 |
| 50 | 35 |
| 100 | 46 |
| 1000 | 59 |
| 10000 | 60 |

Example of sample volume calculation

Let's assume that there are three NM strata in MBA and each strata contains 1000 of inventory items. A stratum is a set of individual inventory items with the same or similar physical characteristics and chemical composition of the nuclear material for statistical sampling. Practically, the strata are usually include: The fuel assemblies from one batch or the same type of fuel assembly, which are identical within the II technical specification for building of critical (subcritical) assemblies; batch of identical products with common passport, etc. In research organizations and organizations with wide range of the used nuclear materials, the number of such strata may be large.

Let's assume that first and second strata in our example are represented by identical inventory items containing highly enriched metal uranium in aluminum shell with numbers drawn with electrosparking pencil. Similarly, the inventory items of the third stratum with low-enriched metal uranium are made. Each inventory item with highly enriched and low-enriched uranium contains 1000 g of 235U. Let's assume that inventory items of the first strata in this inter-balance period was extracted from critical assembly after use in the experiment and placed in tubes working as containers, the pipes were sealed after recording the appropriate inventory data, and inventory items of the second, and third strata in inter-balance period were not used and were sealed in pipes since the previous inventory. Thus, for the first stratum, the probability of detecting a shortage/excess of NM threshold quantity is assumed to be equal to 0.5, and for the second and third strata it is assumed to be equal to 0.25, as the materials of the second and third strata were also under the recording system. This choice of detection probability for each strata is advised, provided that the results of verification the integrity of unibody inventory items and state of the access-control means are documented for documentary confirmation that unauthorized access is not available.

Since the ratio G / x for the first and second strata 8000 / 1000 = 8, and for the third strata 70000 / 1000 = 70, and in all cases G /x <= N (the N value for each strata equals 1000), in all cases we use formula (P2.1) to calculate the sample volume.

According to the formula (P2.1) and parameters G and P for the first stratum, the sample volume will be:

n1 = 1000 [1 - (1 - 0,5)1000/8000] = 1000 [1 - (0,5)1/8] = 1000 х 0,0830 = 83,0.

For the second stratum:

n2 = 1000 [1 - (1 - 0,25) 1000/8000] = 1000 [1 - (0,75)1/8] = 1000 х 0,0353 = 35,3, but by rounding n2 to the nearest larger integer, we get n2 = 36.0.

For the third stratum:

n3 = 1000 [1 - (1 - 0,25) 1000/70000] = 1000 [1 - (0,75)1/70] = 1000 х 0,0041 = 0,4, but by rounding n3 to the nearest larger integer, we get n3 = 1.0.

Thus, shall be be measured in total:

83 + 36 + 1 = 120 inventory items.

Appendix 3
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

ANALYSIS OF SAMPLE DISTRIBUTION CHARACTERISTICS

Table P3.1 shows the values of tk,1-alpha coefficient for the alpha significance level and various degrees of freedom k.

When the random value x is distributed normally with mathematical expectation of a, then from a sample of volume n(x1, x2,...,xn), we can find confidence bounds for "a" as follows: $\left(\overline{x}- \frac{S}{\sqrt{n}} x t\_{k,1-\frac{alpha}{2}}\leq a \leq \overline{x}+ \frac{S}{\sqrt{n}} x t\_{k,1-alpha/2}\right)$, where k = n - 1.

Table P3.1

VALUES OF talpha,k STUDENT'S COEFFICIENT

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$\frac{1-alpha }{k}$$ | 0.80 | 0.90 | 0.95 | 0.98 | 0.99 | 0.995 | 0.999 |
| 23456 | 1.8861.6381.5331.4761.440 | 2.9202.3532.1322.0151.943 | 4.3033.1822.7762.5712.447 | 6.9654.5413.7473.3653.143 | 9.9255.8414.6044.0323.707 | 14.097.4535.5984.7734.317 | 31.6012.928.6106.8695.959 |
|  |  |  |  |  |  |  |  |
| 7891011 | 1.4151.3971.3831.3721.363 | 1.8951.8601.8331.8131.796 | 2.3652.3062.2622.2282.201 | 2.9982.8972.8212.7642.718 | 3.5003.3553.3253.1693.106 | 4.0293.8333.6903.5813.497 | 5.4085.0414.7814.5874.437 |
|  |  |  |  |  |  |  |  |
| 1213141516 | 1.3561.3501.3451.3411.337 | 1.7821.7711.7611.7531.746 | 2.1792.1602.1452.1312.120 | 2.6812.6502.6252.6032.584 | 3.0553.0122.9772.9472.921 | 3.4233.3733.3263.2863.252 | 4.3184.2214.1414.0734.015 |
|  |  |  |  |  |  |  |  |
| 1718192022 | 1.3331.3301.3281.3251.321 | 1.7401.7341.7291.7251.717 | 2.1102.1012.0932.0862.074 | 2.5672.5522.5402.5282.508 | 2.8982.8782.8612.8452.819 | 3.2223.1973.1743.1533.119 | 3.9653.9223.8833.8503.792 |
|  |  |  |  |  |  |  |  |
| 2426283040 | 1.3181.3151.3131.3101.303 | 1.7111.7061.7011.6971.684 | 2.0642.0562.0482.0422.021 | 2.4922.4792.4672.4572.423 | 2.7972.7702.7632.7502.705 | 3.0913.0673.0473.0302.971 | 3.7453.7073.6743.6463.551 |
|  |  |  |  |  |  |  |  |
| 506080100150 | 1.2991.2961.2921.2901.287 | 1.6761.6711.6641.6601.655 | 2.0092.0001.9901.9841.976 | 2.4032.3902.3742.3642.352 | 2.6782.6602.6392.6262.609 | 2.9372.9152.8872.8712.849 | 3.4963.4603.4163.3913.357 |
|  |  |  |  |  |  |  |  |
| 200300500infinity | 1.2861.2841.2831.282 | 1.6531.6501.6481.645 | 1.9721.9681.9651.960 | 2.3452.3392.3342.326 | 2.6012.5922.5862.576 | 2.8392.8282.8202.807 | 3.3403.3233.3103.291 |

Example.

The weight of products is subject to the normal law. There is statistical material for n = 25 samples, for which is found $\overline{x}$ = 8000 g, and s = 600 g. Find confidence limits for mathematical expectation of weight of the entire assembly of samples for alpha = 0.9.

Solution. Using table P3.1 for alpha = 0.90 and k = n - 1 = 24, we find talph, k = 1.711 and determine the confidence bounds for mathematical expectation as follows:

$$7790 g=8000- \frac{600}{\sqrt{24}} х 1,711 \leq a \leq 8000+ \frac{600}{\sqrt{24}} х 1,711=8210 g.$$

Thus, samples with measurement results within the calculated confidence limits can be combined at alpha = 0.9 into a single stratum to perform selective measurements to reduce the amount of measurements when checking the stratum.

Appendix 4
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

RECOMMENDED PROCEDURE FOR APPLYING THE CHECK TOLERANCE

Sample with percentage of Pu-239 equal to the destructive analysis data of 85,06 +/- 0,06 (inventory data) using the U-Pu InSpector gamma-ray spectrometric station with low-energy germanium detector was used. To obtain recording data in accordance with requirements of The Fundamental Rules for Accounting and Control of Nuclear Materials (NP-030-11), the measurement methods shall have metrological certification according to the requirements of standardization documents (measurement error limits are set for a confidence probability value of 95%).

To obtain data on the isotopic composition using U-Pu InSpector gamma-ray spectrometric station with low-energy germanium detector, the spectrum shall be processed with MGA station software. To determine the limits of measurement error, the following method was used: Plutonium and plutonium compounds. Method for measuring the mass fractions of plutonium isotope and americium-241 isotopes in plutonium by gamma-spectrometric method using U-Pu InSpector gamma-ray spectrometer. Method of measurements 223.13.17.104/2006 (limits of measurement error for this method are set for a confidence probability value of 95%). The result obtained with this method was equal to: 85,05 +/- 0,05.

Since all errors are documented and calculated at 95% confidence level, the values of 0.06 and 0.05 shall be reduced by a factor of 1.96 to obtain mean square errors. Thus,

$K = 2,58 \sqrt{sigma\genfrac{}{}{0pt}{}{2}{conf.}+sigma\genfrac{}{}{0pt}{}{2}{inv.}}=2,58\sqrt{(0,06 / 1,96)^{2}+ (0,05 / 1,96)^{2}}=0,10$.

The discrepancy between the recording data and result of the confirming measurements equals 0.01 and lays within the check tolerance of 0.10.

Examples of comparing two sample and average variances

When you need to combine data for two or more NM batches, it is advised to compare sample and average variances. The case of two batches is discussed below. Inventory measurements of two batches, which include n1 = 30 and n2 = 20 inventory items, showed sample values of mathematical expectation and mass variance of the considered nuclide in these inventory items. The values of these quantities are: $\overline{m\_{1}}$ = 40,1 kg; m2 = 40,5 kg; s$\genfrac{}{}{0pt}{}{2}{1}$ = 0,82 kg2; s$\genfrac{}{}{0pt}{}{2}{2}$ = 0,71 kg2. It is required to evaluate significance of differences of the sample variances.

In this example, $F= \frac{s\genfrac{}{}{0pt}{}{2}{1}}{s\genfrac{}{}{0pt}{}{2}{2}} = 1,15 < F\_{0,95}=2,07$ with 29 and 19 degrees of freedom. This shows absence of significant difference in the mass variances of the nuclide subject to inventory in inventory items of the first and second samples. Which is to say, we can assume that the sample data can be combined according to the criterion of equality of variances after checking equality of the average values.

For this purpose let's a combined estimate of the sample variance:

$$s^{2}= \frac{29 х 0,82 + 19 х 0,71}{30 + 20 - 2}= 0,78$$

and value t of the statistics:

$t= \frac{40,1 - 40,5}{\sqrt{0,78 \left(\frac{1}{30} + \frac{1}{20}\right)}}=1,5689$.

Since |t| < tk,1-alpha/2 =2.01 with 48 degrees of freedom, we can assume that $\overline{m\_{1}}$ = $\overline{m\_{2}}$, and the combined estimate value of the sample average equals:

$$\overline{m} = \frac{n\_{1}\overline{m\_{1}}+ n\_{2}\overline{m\_{2}} }{n\_{1}+ n\_{2}}= \frac{30 х 40,1 + 20 х 40,5}{30 + 20}= 40,26 kg$$

Appendix 5
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

EXAMPLES OF CALCULATING sigma o-p AND ALGORITHM FOR EVALUATION THE SIGNIFICANCE OF SYSTEMATIC DISCREPANCIES

Examples of sigmao-p calculation:

Let's assume that the total errors of the scales, reduced to the confidence probability of 0.99, are:

a) the sender +/- 75 g, the receiver +/- 50 g; then

$sigma\_{o-p}= \sqrt{DELTA\genfrac{}{}{0pt}{}{2}{send}+ DELTA\genfrac{}{}{0pt}{}{2}{rec}}= \sqrt{50^{2}+ 75^{2}}= 90,1 g$*;*

b) the sender +/- 0.2%, the recipient +/- 0.3%, and the measured mass is 25 kg; then

$$sigma\_{o-p}= m\sqrt{delta\genfrac{}{}{0pt}{}{2}{send}+ delta\genfrac{}{}{0pt}{}{2}{rec}}= 25 \sqrt{0,002^{2}+ 0,003^{2}}= 0,09 kg$$

c) the sender +/- 0.075 kg, the receiver +/- 0.3%, and the measured mass is 25 kg; then

$$sigma\_{o-p}= \sqrt{delta\genfrac{}{}{0pt}{}{2}{send}+(m delta\_{rec})^{2}}= \sqrt{0,075^{2}+ (25 х 0,003)^{2}}= 0,107 kg$$

Algorithm for evaluating the significance of systematic discrepancies

To check significance of systematic discrepancies between the sender's data msend and receiver's data mrec, it is advised to use the Student's paired t-test. To achieve the said goal calculate:

- the observed pair differences di = (msend)i - (mrec)i;

- the average value of pair differences - $\overline{d}= \sum\_{i=1}^{n}d\_{i}/n$, where n – number of pairs (msend)i, (mrec)i;

- the average square deviation of the average value of paired differences

$S\left(\overline{d}\right)= \sqrt{\frac{SUM (d\_{i}- \overline{d})^{2}}{n (n - 1)}}$;

- observed criterion tn = |d| / S(d).

If condition tn > t(P, f) is met, where t(P, f) is the table value of the Student's coefficient with confidence probability P and number of degrees of freedom f = n - 1, the null hypothesis H0: msend = mrec is rejected.

Example. The table below shows the gross weights of the sender and receiver of fifteen containers containing nuclear material. Acceptable discrepancies based on specification shall not exceed 15 standard units.

Table

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| No. | msend | mrec | d=mrec-mrec | $$d-\overline{d}$$ | $$(d-\overline{d})^{2}$$ | Z |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 282 | 290 | -8 | -0.3 | 0.09 | 286.0 |
| 2 | 347 | 350 | -3 | 4.7 | 22.09 | 348.5 |
| 3 | 286 | 296 | -10 | -2.3 | 5.29 | 291.0 |
| 4 | 319 | 326 | -7 | 0.7 | 0.49 | 322.5 |
| 5 | 337 | 346 | -9 | -1.3 | 1.69 | 341.5 |
| 6 | 290 | 300 | -10 | -2.3 | 5.29 | 295.0 |
| 7 | 301 | 300 | 1 | 8.7 | 75.69 | 300.5 |
| 8 | 314 | 323 | -9 | -1.3 | 1.69 | 318.5 |
| 9 | 373 | 380 | -7 | 0.7 | 0.49 | 376.5 |
| 10 | 236 | 246 | -10 | -2.3 | 5.29 | 241.0 |
| 11 | 361 | 370 | -9 | -1.3 | 1.69 | 365.5 |
| 12 | 333 | 340 | -7 | 0.7 | 0.49 | 336.5 |
| 13 | 292 | 302 | -10 | -2.3 | 5.29 | 297.0 |
| 14 | 340 | 347 | -7 | 0.7 | 0.49 | 343.5 |
| 15 | 320 | 330 | -10 | -2.3 | 5.29 | 325.0 |
| SUM | 4731 | 4846 | -115 | 0.5 | 131.35 | 4788.5 |

The table shows that all observed discrepancies do not exceed the value of acceptable discrepancies, but almost all discrepancies have one sign, which indicates the presence of non-excluded systematic errors in data of the sender or (and) receiver.

Let's check importance of systematic discrepancies.

Based on data in columns 2 and 3, we calculate di = (msend)i - (mrec)i and complete column 5.

Based on the data in column 4, we calculate $\overline{d}= \sum\_{i=1}^{n}d\_{i} / n = -115 / 15= -7,666(6) \~=-7,7 $

Based on data in column 4 we calculate (di - $\overline{d}$) and (di - $\overline{d}$)2, complete columns 5 and 6 calculate $S\left(\overline{d}\right)= \sqrt{\frac{SUM (d\_{i}- \overline{d})^{2}}{n(n-1)}} = \sqrt{\frac{131,35}{15 (15-1)}}=0,791$.

Calculate the observed criterion tn = |$\overline{d}$|$\overline{d}$ / S() = |-7,7| / 0,791 = 9,7362.

The critical value of the Student's coefficient with confidence probability of 0.99 and number of degrees of freedom 14 equals 2.9768.

Since tn = 9.7362 > t(0.9914) = 2.9768 condition is met, the null hypothesis H0: msend = mrec is rejected. Therefore, there are significant systematic discrepancies between the sender's and receiver's data, and it is advised to make calculations based on the average Z values shown in column 7 of the table, until the reasons for the discrepancies are clarified.

Appendix 6
 to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

ADDITIVE AND MULTIPLICATIVE MODEL OF MEASUREMENT ERROR.

RELATION TO THE ABSOLUTE AND RELATIVE MEASUREMENT ERROR

P6.1. Absolute measurement error

By definition, the absolute error is

DELTA = XMEAS - XTRUE, (P6.1)

where:

XMEAS - measured value;

XTRUE - true value.

DELTA absolute error is expressed in units of the measured value. The value of given DELTA error for method of measurement or measuring instruments shall be unchanged throughout the measured (given) range. For example, DELTA = 0.25 grams for scales means that this declared error remains constant over the entire measured range.

Weighing of 1 kg is performed with an error of 0.25 g, weighing of 10 kg is also performed with an error of 0.25 g.

The graph of dependence of the measurement error on the measured value is shown in Fig. P6.1.

 DELTA (gram)

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 0 X (gram)

Fig. P6.1. Dependence of the measurement error on the measured value

In general, the absolute error value does not depend on value of the measured value (additivity property). In practice, the measuring instrument manufacturer can set different absolute error values for different ranges of measured values (for example: from 1 to 100 grams - 0.5 grams, and from 100 to 500 grams - 1 gram).

P6.2. Relative measurement error

By definition, the relative error is

$delta= \frac{DELTA}{Х\_{true}}$. (P6.2)

The relative error represents a dimensionless value, and its values can be expressed in a percentage form.

The value of the given delta error for the method of measurement or measuring instrument is unchanged over the entire measured (given) range. For example, for scales Delta = 0.05% means that this declared error remains constant over the entire measured range. Weighing of 1 kg is performed with an error of 0.05% that is 0.5 g, weighing of 10 kg is also performed with an error of 0.05% that is 5 g. In this case, the absolute error value is directly proportional to the measured value (multiplicative property).

The graph of dependence of the measurement error on the measured value is shown in Fig. P6.2.

 DELTA (gram)

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 0 X (gram)

Fig. P6.2. Dependence of the measurement error on the measured value

P6.3. Additive model of measurement error, relation to the absolute measurement error

The absolute measurement error is structured and represented as the sum of systematic and random components of the error:

DELTA = S + R, (P6.3)

where:

S - error systematic component, has dimension of the measured value;

R is a random component of the error and has dimension of the measured value.

By substituting the right part of expression (P6.3) in (P6.1) and making manipulations, we get:

XMEAS = XTRUE + S + R. (P6.4)

Expression (P6.4) is an additive model of measurement error. This is another record of the absolute measurement error considering discrimination of systematic and random components.

Property of the model (P6.4) from statistical point of view:

XTRUE - deterministic value;

S - random value that usually obeys the normal distribution law N(0, sigmaS);

R - random value that usually obeys the normal distribution law N(0, sigmaR).

The error variance from (P6.4):

$sigma^{2} \left(Х\_{meas}-Х\_{true}\right)= sigma\genfrac{}{}{0pt}{}{2}{S}+sigma\genfrac{}{}{0pt}{}{2}{R}$*,* (P6.5)

where:

SigmaS - mean square deviation of the error systematic component (has dimension of the measured value);

SigmaR - mean square deviation of the error random component (has dimension of the measured value).

Values of sigmaS and sigmaR shall be calculated from the known interval error estimates. Values of sigmaS and sigmaR shall be used for calculating of sigma$\genfrac{}{}{0pt}{}{2}{ID}$.

P6.4. The multiplicative model of measurement errors, relation to the relative measurement error

The relative measurement error is structured and represented as the sum of systematic and random error components:

delta = S + R, (P6.6)

where:

S - systematic error component, dimensionless value;

R - random component of the error, dimensionless value.

By substituting the right part of expression (P6.6) in (P6.2) and making manipulations, we get:

XMEAS = XTRUE (1 + S + R). (P6.7)

Expression (P6.7) is a multiplicative model of the measurement error. This is another record of the relative measurement error considering discrimination of systematic and random components.

Property of the model (P6.7) from statistical point of view:

XTRUE - deterministic value;

S - random value that usually obeys the normal distribution law N(0, sigmaS);

R - random value that usually obeys the normal distribution law N(0, SigmaR)

The error variance from (P6.7):

sigma2 (XMEAS - XTRUE) = X$\genfrac{}{}{0pt}{}{2}{TRUE}$ sigma$\genfrac{}{}{0pt}{}{2}{S}$ + X$\genfrac{}{}{0pt}{}{2}{TRUE}$ sigma$\genfrac{}{}{0pt}{}{2}{R}$ , (P6.8)

where:

SigmaS - mean square deviation of the systematic error component (dimensionless value);

SigmaR - mean square deviation of the random error component (dimensionless value).

Values of sigmaS and sigmaR shall be calculated from the known interval error estimates. Values of sigmaS and sigmaR shall be used for calculating of sigma$\genfrac{}{}{0pt}{}{2}{ID}$.

P6.5. Mixed model and reduced error

Often, additive or multiplicative models do not describe the real errors. In some parts of the measurement range, the model may have one character, in other parts it can differ. In such cases, a mixed error model shall be used. For measuring instruments, the error model is defined during metrological certification.

It is generally assumed that the error model can be considered additive for measuring instrument in part of the range extending up to 1% of its limit. This assumption makes it possible to significantly simplify error estimates providing sufficient degree of correctness.

To describe error of measurement, they often use reduced error, which equals to the maximum value of error over the range attributed to the measurement limits (upper limit of scale). Based on reduced error, you can estimate the maximum error in the range regardless of its model.

Appendix 7
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

CHARACTERISTICS OF THE MEASUREMENT ERROR <\*>

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<\*> Statistical methods for monitoring the quality of measurements and analyzing inventory differences. The methodological materials for the course. 4. The Training Center for Accounting and Control of Nuclear Materials. Obninsk, SSC, RF - Institute of Physics and Energy, 2010.

Table P7.1

CHARACTERISTICS OF THE NUCLEAR MATERIAL MEASUREMENT ERROR

Name of material balance area: Tablet production

|  |  |  |  |
| --- | --- | --- | --- |
| Group (stratum) of nuclear material | Method of measurement  | Random error component | Systematic error component |
| DELTAR | deltaR | sigmaR | THETA (DELTA)S | THETA (delta) S | sigma S |
| 1. Uranium dioxide powder 4,4%
 | 1. Weighing Mettler KE 1500 | - | - | 0,0005 rel. unit |  | - | 0,0005 rel. unit |
| 2. Mass fraction of U OST 95 175-90  | - | 0.0014 | 0,00071 rel. unit |  | 0.0016 | 0,0008 rel. unit |
| 3. Mass fraction of U$\frac{235}{OCT}$ 95 446-84 | - | 0.0075 | 0,0038 rel. unit |  | 0.0042 | 0,0021 rel. unit |
| 1. Grinding waste 4,4%
 | 1. Weighing Mettler KB-60s | - | - | 0,0005 rel. unit | - | - | 0,0005 rel. unit |
| 2. Mass fraction of U TU 002.46-82 | - | 0.0071 | 0,00036 rel. unit | - | 0.0081 | 0,0041 rel. unit |
| 3. Mass fraction of U$\frac{235}{OCT}$ 95 446-84 | - | 0.0075 | 0,0038 rel. unit | - | 0.0042 | 0,0021 rel. unit |
| 1. Tablets 4,4%
 | 1. Weighing Mettler KB-60s | - | - | 0,0005 rel. unit | - | - | 0,0005 rel. unit |
| 2. Mass fraction of U OST 95 175-90 | - | 0.0014 | 0,00071 rel. unit | - | 0.0016 | 0,0008 rel. unit |
| 3. Mass fraction of U$\frac{235}{OCT}$ 95 446-84 | - | 0.0075 | 0,0038 rel. unit | - | 0.0042 | 0,0021 rel. unit |
| 1. Sintered waste 4,4%
 | 1. Weighing Mettler KB-60s | - | - | 0,0005 rel. unit | - | - | 0,0005 rel. unit |
| 2. Mass fraction of U OST 95 175-90 | - | 0.0014 | 0,00071 rel. unit | - | 0.0016 | 0,0008 rel. unit |
| 3. Mass fraction of U$\frac{235}{OCT}$ 95 446-84 | - | 0.0075 | 0,0038 rel. unit | - | 0.0042 | 0,0021 rel. unit |

Appendix 8
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

TABLES OF MATERIAL MOVEMENT IN MATERIAL BALANCE AREA

Table P8.1

TABLE OF MATERIAL MOVEMENT IN FUEL ELEMENT PRODUCTION MATERIAL BALANCE AREA

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Name of material | Batch number | Number of inventory items | Gross weight | Tare weight | Net weight | Mass fraction of element | Mass of element | Isotope mass fraction | Isotope mass |
| Previous physical inventory (AASL) |
| UO2 tablets  | Tab05Tab07Tab10Tab12\_\_\_\_\_Tab15 | 7539174553 |  |  |  |  |  |  |  |
| FE | FE07FE09FE11 | 1358537 |  |  |  |  |  |  |  |
| Reject items (tablets) | Tab07dTab10d | 23 |  |  |  |  |  |  |  |
| Sweepings | Sw01 | 2 |  |  |  |  |  |  |  |
| Supplies and other increases in nuclear material in material balance area (INC) |
| UO2 tablets  | Tab16Tab17\_\_\_\_\_Tab18Tab19Tab20\_\_\_\_\_Tab21Tab122\_\_\_\_\_ | 205203200203229211201 |  |  |  |  |  |  |  |
| Forwarding and other reduction of nuclear material from material balance area (DEC) |
| Rejected items (tablets) | Tab07dTab10dTab35dTab36dTab37d | 23573 |  |  |  |  |  |  |  |
| FE | FE07FE09FE11FE15FE17 |  |  |  |  |  |  |  |  |
| Sweepings | Sw01Sw02 | 24 |  |  |  |  |  |  |  |
| Current physical inventory (AASL) |
| UO2 tablets  | Tab12\_\_\_\_\_Tab17\_\_\_\_\_Tab20\_\_\_\_\_Tab22\_\_\_\_\_ | 45532951 |  |  |  |  |  |  |  |
| FE | FE07FE09FE15 | 15235 |  |  |  |  |  |  |  |
| Rejected items - tab. | - | 0 |  |  |  |  |  |  |  |
| Sweepings | Sw03 | 2 |  |  |  |  |  |  |  |

Table P8.2

GROUP (STRATUM) OF FUEL ELEMENT

Name of material balance area: FUEL ELEMENT PRODUCTION

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Name of material | Batch number | Number of inventory items | Gross weight | Tare weight | Net weight | Mass fraction of element | Mass of element | Isotope mass fraction | Isotope mass |
| Previous physical inventory (AASL) |
| FE | FE07FE09FE11 | 1358537 |  |  |  |  |  |  |  |
| Supplies and other increases in nuclear material in material balance area (INC) |
| FE | - | 0 |  |  |  |  |  |  |  |
| Forwarding and other reduction of nuclear material from material balance area (DEC) |
| FE | FE07FE09FE11FE15FE17 | 120833758115 |  |  |  |  |  |  |  |
| Current physical inventory (AASL) |
| FE | FE07FE09FE15 | 15235 |  |  |  |  |  |  |  |

Table P8.3

GROUP (STRATUM) OF SWEEPINGS

Name of material balance area: FUEL ELEMENT PRODUCTION

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Name of material | Batch number | Number of inventory items | Gross weight | Tare weight | Net weight | Mass fraction of element | Mass of element | Isotope mass fraction | Isotope mass |
| Previous physical inventory (AASL) |
| Sweepings | Sw01 | 2 |  |  |  |  |  |  |  |
| Supplies and other increases in nuclear material in material balance area (INC) |
| Sweepings |  | 0 |  |  |  |  |  |  |  |
| Forwarding and other reduction of nuclear material from material balance area (DEC) |
| Sweepings | Sw01Sw02 | 24 |  |  |  |  |  |  |  |
| Current physical inventory (AASL) |
| Sweepings | Sw03 | 2 |  |  |  |  |  |  |  |

Table P8.4

GROUP (STRATUM) OF TABLETS (REJECTED)

Name of material balance area: FUEL ELEMENT PRODUCTION

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Name of material | Batch number | Number of inventory items | Gross weight | Tare weight | Net weight | Elem. mass fr.  | Mass of element | Isotope mass fr. | Isotope mass |
| Previous physical inventory (AASL) |
| UO2 tablets  | Tab05Tab07Tab10Tab12\_\_\_\_\_Tab15 | 7539174553 |  |  |  |  |  |  |  |
| Reject items (tablets) | Tab07d\_\_\_\_\_\_Tab10d\_\_\_\_\_\_ | 23 |  |  |  |  |  |  |  |
| Supplies and other increases in nuclear material in material balance area (INC) |
| UO2 tablets  | Tab16Tab17\_\_\_\_\_Tab18Tab19Tab20\_\_\_\_\_Tab21Tab122\_\_\_\_\_ | 205203200203229211201 |  |  |  |  |  |  |  |
| Forwarding and other reduction of nuclear material from material balance area (DEC) |
| Reject items (tablets) | Tab07d\_\_\_\_\_\_Tab10d\_\_\_\_\_\_Tab35dTab36dTab37d | 23573 |  |  |  |  |  |  |  |
| Current physical inventory (AASL) |
| UO2 tablets  | Tab12\_\_\_\_\_Tab17\_\_\_\_\_Tab20\_\_\_\_\_Tab22\_\_\_\_\_ | 45532951 |  |  |  |  |  |  |  |
| Rejected items - tab. | - | 0 |  |  |  |  |  |  |  |

Appendix 9
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

EXAMPLE OF EXCLUSION OF PAIRED RECORDS

The group (stratum) of tablets of fuel element production is presented in table P8.4 of Appendix 8 to this Provision.

In this example, the batch of Tab12 (45 containers) is present in previous (IQ) and current inventory (AQ), the batches of Tab17, Tab20, Tab22 entered the material balance area (INC) and were partially processed, i.e. assembled in fuel elements, the rest of containers of these batches are present in the current inventory (AQ). Information on containers in these parties represents "paired inventory records". The rest of tablet batches were assembled in fuel elements.

Tab07d, Tab10 (defective tablets) batches from the previous inventory (IQ) were sent from the material balance area (DEC). The inventory items of these batches represent "paired inventory records".

After exclusion of "paired inventory records", this group (stratum) is presented in table P9.1.

Table P9.1

GROUP (STRATA) OF TABLETS (EXCLUDED "PAIRED INVENTORY RECORDS")

Name of material balance area: FUEL ELEMENT PRODUCTION

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Name of material | Batch number | Number of inventory items | Gross weight | Tare weight | Net weight | Elem. mass fr.  | Mass of element | Isotope mass fr. | Isotope mass |
| Previous physical inventory (AASL) |
| UO2 tablets  | Tab05Tab07Tab10Tab15 | 75391753 |  |  |  |  |  |  |  |
| Reject items (tablets) | Tab07d\_\_\_\_\_\_Tab10d\_\_\_\_\_\_ | 23 |  |  |  |  |  |  |  |
| Supplies and other increases in nuclear material in material balance area (INC) |
| UO2 tablets  | Tab16Tab17\_\_\_\_\_Tab18Tab19Tab20\_\_\_\_\_Tab21Tab122\_\_\_\_\_ | 205150200203200211150 |  |  |  |  |  |  |  |
| Forwarding and other reduction of nuclear material from material balance area (DEC) |
| Rejected items (tablets) | Tab35dTab36dTab37d | 573 |  |  |  |  |  |  |  |
| Current physical inventory (AASL) |
| UO2 tablets  |  |  |  |  |  |  |  |  |  |

Appendix 10
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

EVALUATION COMPONENT OF THE VARIANCE, WHICH DECREASES THE CALCULATED sigma$\frac{2}{ID}$ ACCORDING TO THE AVERAGING METHOD OVER THE STREAM

If we denote the sum of mass of the element (isotope) in n II batch as MNM, then the average mass of nuclear material in one inventory item will be $\frac{M\_{NM}}{n}$.

To calculate sigma$\frac{2}{ID}$, this average mass value shall be substituted for the summation sign, and the analytical expressions will contain the value of mass of the entire batch, and this reduces amount of calculations.

If we find difference between the analytical expressions for calculating sigma$\frac{2}{ID}$ using method without averaging over the stream and method with averaging over the stream (for multiplicative models of measurement error), then the component that decreases the calculated sigma$\frac{2}{ID}$ estimate using method of averaging over the stream is:

$V\_{(-)}= \left[\sum\_{i=1}^{n}(m\_{NMi})^{2}- \frac{M\genfrac{}{}{0pt}{}{2}{NM}}{n}\right] \left(sigma\genfrac{}{}{0pt}{}{2}{RM}+sigma\genfrac{}{}{0pt}{}{2}{RA}\right)$, (P10.1)

where:

MNMi - mass of nuclear material in each batch of inventory item;

MNM - mass of nuclear material in the entire batch;

n - number of inventory items in the batch;

SigmaRM - mean square deviation of the random component for the weighing error;

SigmaRA - mean square deviation of the random component for the MM error of destructive analysis method.

Example:

Let there be a batch of nuclear material including 50 inventory items, and their measured mass values are presented below. Measurement error: SigmaRM = 0.05%, SigmaRA = 0.1%.

41.39 40.96 40.66 40.74 41.6 40.87 41.26 41.46 40.77 41.29

41.85 41.22 40.83 40.44 41.24 40.96 41.3 41.87 41.32 41.09

42.33 40.99 41.5 41.02 41.82 40.36 40.9 41.04 40.85 41.34

41.72 41.62 41.92 39.74 41.36 41.08 41.14 40.88 41.62 41.12

40.93 42.2 41.14 41.33 40.7 41.35 41.2 41.1 41.31 41.58

Average mass of nuclear material in one inventory item = 41.206.

The component that decreases the estimate using method of averaging over the stream is:

V(-) = 1,3 х 10-5.

Appendix 11
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

ANALYTICAL EXPRESSIONS FOR CALCULATION OF sigma$\frac{2}{ID}$ WITHOUT AVERAGING OVER THE STREAM.

For calculating sigma$\frac{2}{ID}$, the analytical expressions for multiplicative models of measurement errors are considered as the most frequently used in international practice.

P11.1. Contribution of the systematic error components in the sigma$\frac{2}{ID}$

Contribution of the systematic measurement error component for one of the measurement methods (weighing, volume measurement, measurement method for destructive and non-destructive testing methods) to sigma$\frac{2}{ID}$ in one of the groups (strata) STRj can be calculated (without averaging over the stream):

$V\_{Sj}= \left[\sum\_{i belongs STR\_{j}}^{}sgn \left(m\_{i}\right)m\_{NMi}\right]^{2}sigma\genfrac{}{}{0pt}{}{2}{Sj}$*,* (P11.1)

where:

sgn (mi) = +1 for mi from IQ, INC - terms of ID equation,

sgn(mi) = -1 for mi from AQ, DEC - terms of ID equation.

sigmaSj - mean square deviation of the systematic error component of one of the measurement methods,

This approach is valid, for example, when weighing the entire nuclear material that makes up the terms of balance equation with the same scales without recalibration.

Contribution to the variance of the systematic weighting error component will be denoted with index "M": VSMj and sigmaSMj. When measuring volume, replace "M" with "V" in the notations. Contribution to the variance of the systematic MM error component of destructive and non-destructive testing methods will be denoted with index "A": VSAj and sigmaSAj.

When weighing and determining the element (isotope) mass fraction, the following approach can be used.

When calculating the inventory difference by element:

mNMi = (mбi - mti) Ci, (P11.2)

where:

mбi - gross weight;

mti - tare weight;

Ci - mass fraction of the element.

For calculating ID by isotope:

mNMi = (mбi - mti) Ci Cii, (P11.3)

where Sii - isotope mass fraction.

P11.2. Contribution of the random error components to sigma$\frac{2}{ID}$

P11.2.1. Contribution of the random component for the weighting error to sigma$\frac{2}{ID}$ in one of the groups (struta) STRj (without averaging over the stream):

$V\_{RMj}= \sum\_{i belongs STR\_{j}}^{}\frac{\left(m\genfrac{}{}{0pt}{}{2}{бi}+m\genfrac{}{}{0pt}{}{2}{ti}\right)C\genfrac{}{}{0pt}{}{2}{i}}{n\_{i}} sigma\genfrac{}{}{0pt}{}{2}{RMj}$, (P11.4)

where:

ni - number of weighings of the i-th container in the j-th group (stratum) (company's instructions determine requirement to record the average measurement result by ni results of measuring of identical volume), in most cases ni = 1;

sigmaRMj - mean square deviation of the random component for the weighting error in the j-th group (stratum);

Ci - mass fraction of the element.

When calculating sigma$\frac{2}{ID}$ by isotope, instead of Ci substitute Ci х Ciiin (P11.4).

Expression (P11.4) is used for calculating sigma$\frac{2}{ID}$ without averaging over the stream, where mbi, mti represent individual gross and tare masses of the i-th container in the j-th group (stratum).

P11.2.2. Contribution of the random component of the volume measurement error to Sigma$\frac{2}{ID}$ in one of the groups (strata) (without averaging over the stream):

$V\_{RVj}= \sum\_{i belongs STR\_{j}}^{}\frac{\left(v\_{i} c\_{i}\right)^{2}}{n\_{i}} sigma\genfrac{}{}{0pt}{}{2}{RVj}$, (P11.5)

where:

ni - number of measurements of the i-th volume (company's instructions determine requirement to record the average measurement result by ni results of measuring of identical volume), in most cases ni = 1;

sigmaRVj - mean square deviation of the random component for the volume measurement error in the j-th group (stratum);

Ci - mass fraction of the element.

When calculating sigma$\frac{2}{ID}$ by isotope, instead of Ci, substitute Ci x Cii in (P11.5).

Expression (P11.5) is used for calculating sigma$\frac{2}{ID}$ without averaging over the stream, where Vi is individual volume measurements in the j-th group (stratum).

P11.2.3. Contribution of the random MM error component of one of the methods of destructive and non-destructive testing in the j-th group (stratum) (without averaging over the stream) to Sigma$\frac{2}{ID}$:

$V\_{RAj}= \sum\_{i=1}^{R}\frac{1}{p\_{i}} \sum\_{k=1}^{N\_{i}}\left[(m\_{бik}- m\_{tik})C\_{i}\right]^{2} sigma\genfrac{}{}{0pt}{}{2}{RAj}$, (P11.6)

where:

R - number of batches of nuclear material, where mass fraction of the element (isotope) is calculated using measurement method for the j-th group (stratum);

Ni - number of containers in the batch;

mbik, mtik - gross weight, tare of the k-th container from the i-th batch;

pi - number of samples per batch.

Ci - mass fraction of the element.

sigmaRAj - mean square deviation of the random component for the MM error for the j-th group (stratum).

When calculating sigma$\frac{2}{ID}$ by isotope, instead of Ci, substitute Ci x Cii in (P11.6).

Appendix 12
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

EXAMPLE OF CALCULATING OF THE TOTAL MASS OF MATERIAL

Table P12.1 shows the inventory data from the previous example for FE tablet batches and results of calculating the total MNMj mass:

MNMS (syst) = 6356,6793 + 47892,794 - 0 - 456,18745 = 53793,286 kg (for element)

MNMS (syst) = 185,63364 + 1413,7849 - 0 - 13,463192 = 1585,9553 kg (for isotop)

MNMR (ran) = 6356,6793 + 47892,794 + 0 + 456,18745 = 54705,661 kg (for element)

MNMR (ran) = 185,63364 + 1413,7849 + 0 + 13,463192 = 1612,8817 kg (for isotope)

Table P12.1

THE TOTAL MASS OF MNMs (SYST) AND MNMR (RAN), THE "PAIRED INVENTORY RECORDS" ARE EXCLUDED"

Name of material balance area: FUEL ELEMENT PRODUCTION

|  |
| --- |
| Group (strata) of tablets (excluded "paired inventory records") |
| Term of ID equation  | Batch | Number of inventory items | Net weight, kg | MU, fract.% | Mass MU, kg | U$\frac{M}{235}$, fract.% | Mass MU235, kg |
| IQ | Tab05Tab07Tab10Tab15 | 75391753 | 3098.251615.38706.011859.40 | 87.387.587.487.2 | 2704.77231413.4575617.052741621.3968 | 2.912.872.952.97 | 78.70887440.5662318.20305648.155485 |
| Sum |  |  |  | 6356.6793 |  | 185.63364 |
| INC | Tab16Tab17\_\_\_\_\_Tab18Tab19Tab20\_\_\_\_\_Tab21Tab22\_\_\_\_\_ | 205150200203200211150 | 8513.656202.508286.008426.538284.458754.396234.62 | 87.687.787.587.487.387.887.6 | 7457.95745439.59257250.257364.78727232.32497686.35445461.5271 | 2.942.952.982.932.952.972.94 | 219.26395160.46798216.05745215.78826213.35358228.28473160.5689 |
| Sum |  |  |  | 47892.794 |  | 1413.7849 |
| DEC | Tab35dTab36dTab37d | 573 | 172.85242.9105.45 | 87.587.687.4 | 151.24375212.780492.1633 | 2.982.942.93 | 4.50706376.25574382.7003847 |
| Sum | 0 |  |  | 456.18745 |  | 13.463192 |
| AQ | - | 0 |  |  | 0 |  | 0 |
|  | Sum | 0 |  |  | 0 |  | 0 |
| MNMS (SYST) |  |  |  |  | 53793.286 |  | 1585.9553 |
| MNMR (RAN) |  |  |  |  | 54705.661 |  | 1612.8817 |

Appendix 13
to the Provision on use of mathematical statistics methods for record and control of nuclear materials, approved by the Order of the Federal Environmental, Industrial and Nuclear Supervision Service,
September 14, 2011. No. 535

EXAMPLE OF CALCULATING THE CONTRIBUTION OF SYSTEMATIC AND RANDOM COMPONENTS OF THE GROUP (STRATA) ERRORS TO sigma$\frac{2}{ID}$

Table P13.1 shows results of calculating the contribution of the systematic and random components of the group (strata) errors to Sigma$\frac{2}{ID}$ according to Annex 12 of this Provision.

Table P13.1

CONTRIBUTION OF THE SYSTEMATIC AND RANDOM COMPONENTS FOR THE GROUP (STRATA) OF TABLETS TO sigma$\frac{2}{ID}$

Name of material balance area: FUEL ELEMENT PRODUCTION

|  |
| --- |
| Group (strata) of tablets (excluded "paired inventory records") |
| Term of ID equation  | Batch | Number of inventory items | Net weight, kg | MU, fract.% | Mass MU, kg | U$\frac{M}{235}$, fract.% | Mass MU235, kg |
| IQ | Tab05Tab07Tab10Tab15 | 75391753 | 3098.251615.38706.011859.40 | 87.387.587.487.2 | 2704.77231413.4575617.052741621.3968 | 2.912.872.952.97 | 78.70887440.5662318.20305648.155485 |
| Sum |  |  |  | 6356.6793 |  | 185.63364 |
| INC | Tab16Tab17\_\_\_\_\_Tab18Tab19Tab20\_\_\_\_\_Tab21Tab22\_\_\_\_\_ | 205150200203200211150 | 8513.656202.508286.008426.538284.458754.396234.62 | 87.687.787.587.487.387.887.6 | 7457.95745439.59257250.257364.78727232.32497686.35445461.5271 | 2.942.952.982.932.952.972.94 | 219.26395160.46798216.05745215.78826213.35358228.28473160.5689 |
| Sum |  |  |  | 47892.794 |  | 1413.7849 |
| AQ | - | 0 |  |  | 0 |  |  |
|  | Sum | 0 |  |  | 0 |  |  |
| DEC | Tab35dTab36dTab37d | 573 | 172.85242.9105.45 | 87.587.687.4 | 151.24375212.780492.1633 | 2.982.942.93 | 4.50706376.25574382.7003847 |
| Sum | 0 |  |  | 456.18745 |  | 13.463192 |
| Contribution of the systematic error components to sigma$\frac{2}{ID}$ |
|  |  |  |  |  | Element |  | Isotope |
| MNMS (SYST) |  |  |  |  | 53793.286 |  | 1585.9553 |
| Weighing of sigmaS |  |  |  |  | 0,05% |  | 0,05% |
| VMS |  |  |  |  | 723.4294 |  | 0.62881 |
| Evaluation of sigmaS |  |  |  |  | 0,1% |  | 0,08% |
| VAS |  |  |  |  | 2893.7176 |  | 1.60976 |
| VS SUM = VMS + VAS |  |  |  |  | 3617.147 |  | 2.23857 |
| Contribution of the random error components to sigma$\frac{2}{ID}$ |
| MNMR (RAN) |  |  |  |  | 54705.661 |  | 1612.8817 |
| Weighing of sigma MR |  |  |  |  | 0,07% |  | 0,07% |
| Number of containers |  |  |  |  | 1518 |  | 1518 |
| VMR |  |  |  |  | 0.966 |  | 0.00084 |
| Evaluation of sigmaAR |  |  |  |  | 0,15% |  | 0,1% |
| Number of evaluations |  |  |  |  | 547 |  | 11 |
| VAR |  |  |  |  | 12.31005 |  | 0.23649 |
| VR SUM = VMR + VAR |  |  |  |  | 13.27605 |  | 0.23733 |

For this example, each container is weighed one time, so Nj corresponds to the total number of containers (1518). The mass fraction of the element and isotope was determined in a different material balance area. On average, to determine the mass fraction of the element- 2 samples per 200 kg of nuclear material were taken, and to determine the mass fraction of the isotope - 2 samples per 300 kg of nuclear material were taken. Each sample was evaluated one time, so the number of evaluations to determine the mass fraction of the element equals (54705.661 / 200) x 2 = 547, the number of evaluations to determine the mass fraction of the isotope equals (1612.8817 / 300) x 2 = 11.

For weighing, the mean square deviation of the systematic error component sigmaSM = 0.05% (corresponds to international ITV target values - 2000). For the destructive testing method, the systematic error component in determining the mass fraction of the element sigmaSA = 0.1%, and determining the mass fraction of the isotope sigmaSA = 0.08%.

Contribution of еsystematic error components in sigma$\frac{2}{ID}$:

- VSM - weighing (723.4294 kg2 - by element, 0.62881 kg2 – by isotope);

- VSA - destructive evaluation test (2893.7176 kg2 - by element, 1.60976 kg2 - by isotope);

VS SUMj  - the total contribution of systematic error components to sigmaID of the j-th group (stratum) (3617.147 kg2 - by element, 2.23857 kg2 – by isotope).

For weighing, the mean square deviation of the random error component sigmaRM = 0.05% (corresponds to international ITV target values - 2000).

For the destructive testing method, the random error component in determining the mass fraction of the element SigmaRA = 0.15%, and determining the mass fraction of the isotope SigmaRA = 0.1%.

Contribution of the random error components to sigma$\frac{2}{ID}$:

- VRM - weighing (0.966 kg2 - by element; 0.00084 kg2 – by isotope);

- VRA - destructive evaluation test (12.31005 kg2 - by element; 1.60976 kg2 - by isotope);

- VR SUMj - the total contribution of the systematic error components of the j-th group (stratum) to Sigma$\frac{2}{ID}$ (3617.147 kg2- by element; 0.23649 kg2 - by isotope).